Environmental Policy Effectiveness under Alternative Monetary Policy Regimes in Small Open Economies

Wenbo He [*] Guangzhi Ye[†]

April 10, 2026

Abstract

This paper studies the interaction between environmental policy and monetary policy in a small open economy using a two-sector Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model. The economy features a brown sector and a green sector and incorporates two widely used environmental policy instruments: a carbon tax and a cap-and-trade system. We embed this framework in a small open economy with alternative monetary policy and exchange rate regimes to examine how macroeconomic policy design affects the transmission and effectiveness of environmental regulation. We first characterize steady-state outcomes under varying degrees of carbon tax rates and cap-and-trade stringency, allowing for static comparisons across environmental policy instruments. We then study the dynamic responses of macroeconomic variables to productivity and monetary policy shocks under each environmental policy regime across four small open economy configurations. We conduct both static and dynamic welfare analyses to compare carbon taxation and cap-and-trade in open economies.

JEL Classification: E42, F41, Q56

Keywords: Small Open Economies, Green Transitions, Carbon Taxes, Exchange Rate Regimes, Monetary Policy

---

[*] Division of Economics, Nanyang Technological University, (wenbo001@e.ntu.edu.sg)

[†] Division of Economics, Nanyang Technological University, (guangzhi.ye@ntu.edu.sg)

Financial support from NTU - Start Up Grant (SUG-Dec 2022) at Nanyang Technological University is gratefully acknowledged.

1 Introduction

Growing evidence shows that climate change poses substantial risks to both physical and mental health through more frequent extreme weather events, natural disasters, and rising global temperatures (Massazza et al., 2025; Cho and Ackom, 2025). Human activities, particularly the extensive use and combustion of fossil fuels, have long been identified as key drivers of rising carbon dioxide (CO_2) concentrations and global warming (e.g., Al-Ghussain (2019); Fang et al. (2023); Lenton et al. (2023)). Carbon emissions are therefore widely recognized as a primary contributor to climate change (Gatti et al., 2021).

Given these mounting risks, reducing greenhouse gas emissions has become an urgent global priority. The United Nations has repeatedly called for coordinated international action, culminating in the Paris Agreement in 2015, a landmark commitment to limit global temperature increases and achieve net-zero emissions within this century (UN, 2025, 2026). Consequently, a rapid transition toward a Net Zero economy has emerged as a central policy objective worldwide.

The green transition involves a structural transformation toward sustainable development through the adoption of renewable energy and cleaner production technologies, leading to higher demand for green goods and lower demand for carbon-intensive (brown) goods. While policymakers and academics actively debate the design of green policy instruments to curb emissions (Kolcava, 2023; Wu et al., 2024; Buntaine et al., 2024), important gaps remain. In particular, there is limited comparative evidence on the effectiveness of carbon pricing across different types of small open economies. Moreover, concerns about output losses, transitional dynamics, and welfare implications raise fundamental questions about the trade-offs between emission reductions and economic performance. These issues motivate a systematic comparison between a traditional single-sector (brown) economy and a dual-sector economy with both brown and green production across different types of small open economies characterized by heterogeneous monetary policy regimes—an analysis that remains largely underexplored in the literature.

Building on international climate commitments, governments have adopted market-based environmental instruments, most notably carbon taxes and cap-and-trade systems (Etukudoh et al., 2024; Kosnik, 2018; Zhou et al., 2022). A carbon tax imposes a fixed charge per unit of emissions, whereas a cap-and-trade system sets a quantitative emissions limit with tradable permits. We incorporate both instruments to examine how carbon pricing affects emissions reduction and the reallocation of demand toward green goods during the green transition.

We develop a small open economy environmental dynamic stochastic general equilibrium (E-DSGE) model to analyze green transitions under four distinct monetary policy regimes, covering the main policy frameworks considered in the literature. The framework applies to economies that conduct monetary policy through interest rate rules and implement carbon pricing as a fiscal instrument. By allowing policy parameters and tax rates to vary across regimes, the model provides a unified structure to examine how monetary policy interacts with carbon pricing. Moreover, we incorporate a cap-and-trade policy by varying the emissions cap and compare its economic and welfare effects with those of carbon pricing across different economies.

The model extends a standard small open economy DSGE framework by introducing heterogeneous brown and green production sectors. A representative household consumes, supplies labor, and invests. On the production side, intermediate firms produce differentiated brown and green inputs, which are processed through retailers and aggregators into sectoral composites. These are combined into a domestic final good and then aggregated with imports into an Armington composite good. Uncovered interest parity holds, and monetary policy follows a Taylor-type rule with regime-dependent parameters to capture alternative policy frameworks.

Using calibrated parameters, we obtain three main sets of findings. First, carbon pricing policies generate a clear green transition in both steady state and dynamics. In steady state, higher carbon taxes or tighter emission caps increase consumption, investment, labor demand, and output in the green sector, while reducing these variables in the brown sector. This pattern holds across all monetary–exchange rate regimes considered. In response to external shocks, such as positive TFP shock or contractionary monetary policy shocks, introducing higher carbon taxes amplifies the expansion of the green sector and deepens the contraction of the brown sector. Stricter environmental policies consistently lower aggregate emissions and brown-sector demand while raising green-sector demand.

Second, monetary–exchange rate regimes differ markedly in their macroeconomic responses under carbon pricing. Under a benchmark carbon tax (e.g., 0.0512), the fixed peg regime exhibits the largest fluctuations in key macroeconomic variables, whereas the managed float regime delivers the most stable responses to aggregate shocks. Inflation-targeting and dual-targeting regimes lie between these two extremes, with inflation targeting generally more volatile in output, exports, and imports than dual targeting. These patterns are robust under both carbon tax and cap-and-trade policies and under TFP and world interest rate shocks. As a robustness check, we also consider alternative tax rates and emissions caps across the four monetary policy regimes. The results confirm the benchmark findings: the managed float regime exhibits more muted and stable responses when environmental regulation becomes stricter, whereas the fixed exchange rate regime generates more volatile adjustments, particularly in price and output variables.

Third, welfare outcomes vary across both environmental instruments and monetary regimes. Cap-and-trade generates smaller welfare losses than carbon taxation across all regimes, consistent with the existing E-DSGE literature (e.g., Annicchiarico and Di Dio (2015); Annicchiarico and Diluiso (2019)). For example, when the carbon tax is calibrated to generate the same emissions reduction as tightening the emissions cap from 100% to 95%, the consumption-equivalent welfare gain under cap-and-trade is approximately 0.302% higher than under the carbon tax. Among monetary frameworks, the managed float regime delivers the lowest welfare losses as carbon pricing becomes more stringent, while the fixed peg regime performs worst. If the carbon tax rate increases from a low level of 0.085 to a medium level of 0.124, equivalent to tightening the emissions cap from 95% to 90%, the managed float regime generates welfare gains that are 0.015% and 0.074% higher than those under the fixed exchange rate regime for the carbon tax policy and the cap-and-trade policy, respectively.Although quantitatively small, the result consistently indicates that the managed float regime mitigates welfare losses associated with stricter environmental regulation. Overall, for small open economies undergoing a green transition, a managed float regime appears more favorable in terms of welfare performance than the alternative monetary–exchange rate regimes considered.

Related Literature. This paper relates to three strands of the existing literature. First, it contributes to the extensive body of research on macroeconomic models of small open economies. Since the seminal contributions of Schmitt-Grohé and Uribe (2003) and Gali and Monacelli (2005), DSGE models for small open economies have become a standard tool in modern macroeconomics. Building on this framework, Adolfson et al. (2008) develop a New Keynesian DSGE model for a small open economy and propose a comprehensive estimation strategy for empirical analysis. More recently, Lozej et al. (2023) employ a small open economy model to study the macroeconomic effects of capital regulation within a monetary union, while Dong et al. (2025) use a small open economy DSGE framework to analyze financial crises, asset price bubbles, and their interactions. Our paper builds on this literature by applying a rich small open economy DSGE model to study green transitions, with a particular focus on the macroeconomic and policy implications of environmental regulation.

Second, this study contributes to the growing literature on the economic consequences of green transitions. A number of recent papers emphasize the role of political economy, perceptions, and financial spillovers in shaping the transition process. For example, Besley and Persson (2025) examine political perceptions of green products and energy, highlighting how public attitudes and social image concerns influence green adoption. Mzoughi et al. (2022) analyze spillover effects from energy sector transitions and show how these can propagate uncertainty in green financial markets. Tyquin et al. (2024) demonstrate that informational framing regarding economic costs and environmental benefits affects households' willingness to adopt green products. In addition, Akhtaruzzaman and Rahman (2024) provide a comprehensive assessment of the downside risks associated with green transitions and conclude that such risks have declined substantially over time. In contrast to this predominantly empirical and survey-based literature, we study green transitions through the joint use of carbon pricing and monetary policy in a structural DSGE framework with heterogeneous producers in a small open economy, a setting that remains largely unexplored in international macroeconomics.

Finally, this paper offers novel theoretical insights into the joint implementation of carbon taxation and monetary policy in small open economies operating under different policy regimes. We evaluate how regime heterogeneity shapes both emission reductions and macroeconomic outcomes, and we carefully characterize the resulting policy trade-offs during the energy transition. Existing work in this area is limited. For instance, Ferrari and Landi (2024) analyze green transition dynamics and identify a potential source of green inflation, but do not consider a small open economy environment or alternative policy regimes. Ploeg et al. (2022) study the distributional effects of carbon tax reforms using survey data on German households, while Bürgisser et al. (2024) examine the effectiveness of green taxation and public support for such policies. Our contribution differs by embedding carbon pricing into a small open economy DSGE model and conducting counterfactual policy experiments that generate rich dynamic responses of both macroeconomic variables and emissions.

The rest of the paper is structured as follows. Section 2 introduces the policy and institutional backgrounds that motivate our study. Section 3 presents our theoretical model and the calibrations used for our numerical analysis. Section 4 shows our comparative statics and impulse response analysis. Section 5 presents the welfare analysis under different policy combinations. Section 6 concludes, discusses limitations, and outlines directions for future research.

2 Institutional and Policy Background

This section outlines the institutional features and policy environments of different small open economies that motivate our modeling framework. In the canonical small open economy setting, domestic policy actions do not affect foreign economies directly, but external factors, such as exchange rate fluctuations and changes in the terms of trade, play a central role in shaping domestic macroeconomic outcomes (Gali and Monacelli, 2005; Christiano and Trabandt, 2011; Schmitt-Grohé and Uribe, 2003). Consequently, the conduct of monetary policy in small open economies is closely tied to the exchange rate regime. As emphasized by Smiech et al. (2024), exchange rate arrangements are often the dominant policy consideration for monetary authorities when stabilizing output and inflation. More broadly, macroeconomic research highlights the importance of exchange rates and the global monetary system for financial stability in open economies (Bocola and Lorenzoni, 2020; Obstfeld and Zhou, 2022; Goldberg and Reed, 2023).

Using a comprehensive classification of exchange rate regimes, Ilzetzki et al. (2021) show that neither pure floating nor strictly fixed exchange rate regimes have become dominant globally. Instead, most countries operate under some form of managed float with varying degrees of policy autonomy. Motivated by this observation, we classify small open economies according to their monetary and exchange rate policy frameworks and focus on four representative regimes in our theoretical analysis: (i) a nearly fixed exchange rate regime, (ii) a managed floating exchange rate regime, (iii) strict inflation targeting, and (iv) dual targeting of inflation and output.

Alongside differences in monetary and exchange rate arrangements, many small open economies have increasingly adopted carbon pricing and green production policies to support the transition toward low-carbon growth. Across the four regimes we consider, real-world policy initiatives provide clear institutional motivation for modeling carbon pricing and green production within a small open economy DSGE framework. In what follows, we briefly describe representative economies corresponding to each policy regime and summarize their approaches to monetary policy and green transition by environmental regulation.

We begin with economies operating under a nearly fixed exchange rate regime. Hong Kong provides a canonical example. According to the Hong Kong Monetary Authority, the primary objective of Hong Kong's monetary policy is to maintain a stable exchange value of the Hong Kong dollar (HKMA, 2025). The local government has set a carbon-neutrality primary goal due to be achieved by 2050, while outlining four major green transition paths along with this policy objective. While Hong Kong has not implemented an explicit carbon tax, the government has actively promoted low-emission technologies, sustainable infrastructure, and the use of clean energy (Hong Kong Government, 2026). These policy efforts motivate our modeling of a dual production structure that distinguishes between carbon-intensive ("brown") and low-emission ("green") sectors with differing production characteristics.

Managed float exchange rate regime is one of the popular policy choices among small open economies around the world. By regulating the exchange rate within a fluctuation band, this regime may provide more export cushion against a change of terms of trade (Irwin, 2025), or be more immune to financial crisis when there is a global shock (Ghosh et al., 2015). Singapore represents a managed floating exchange rate regime with policy bands. Under the Monetary Authority of Singapore's (MAS) framework, monetary policy is conducted through the management of the Singapore dollar against a trade-weighted basket of currencies, with the objective of maintaining price stability and supporting sustainable growth (MAS, 2025). Although the exchange rate is allowed to fluctuate, it is tightly managed within a policy band. Singapore has also been among the early adopters of carbon pricing in the region, introducing a carbon tax in 2019 at 5 Singapore dollars per ton of CO_2 emissions. The tax is scheduled to rise incrementally to 25 Singapore dollars per ton, reflecting a strengthening commitment to green transition policies. In a similar policy manner, Singapore has also participated in international carbon trading market by allowing the local firms to exchange carbon credits, which assists with the firms' green transition by channeling the flow of green credits (National Climate Change Secretariat, 2026).

South Korea exemplifies a strict inflation targeting regime with a flexible exchange rate. The Bank of Korea (BoK) explicitly states that its primary monetary policy objective is price stability, as stipulated in Article 1 of the Bank of Korea

Act (BoK, 2025). Unlike Hong Kong and Singapore, exchange rate management does not feature prominently in Korea's monetary policy mandate. Following Alba et al. (2020), we characterize this framework as a flexible exchange rate regime centered on inflation control. The inflation targeting policy mandate is widely adopted by central banks around the world, aiming to strictly manage the price change. In a small open economy scenario, it is more interesting to note the difference between exchange rate management and inflation control under free float of exchange rate. While South Korea has not introduced an explicit carbon tax, it levies fuel excise taxes that are commonly interpreted as implicit carbon taxes on CO_2 emissions. In 2015, the national government established an emission trading system (ETS) to allow firms operating in South Korea to exchange emission permits under policy quota (International Carbon Action Partnership, 2026). This arrangement features a cap-and-trade policy initiative which governs the trading of emission credits under a national quota guidance.

Finally, we consider economies operating under a dual-target monetary policy regime, where both inflation and output stabilization play a role in policy decisions. Israel provides a representative example. The Bank of Israel states that its primary objective is to maintain price stability and preserve the value of the local currency (BoI, 2025), consistent with an inflation targeting framework. At the same time, the central bank also emphasizes the importance of supporting economic growth and reducing macroeconomic volatility. As documented by Kazinnik and Papell (2021), the Bank of Israel actively intervenes to mitigate economic fluctuations, even in the absence of explicit exchange rate targets. We therefore classify Israel as a free-floating exchange rate economy with inflation as the primary policy objective and output stabilization as a secondary consideration. In terms of environmental policy, Israel introduced a carbon tax in 2023, with a plan to gradually increase carbon prices through 2028.

These institutional observations directly inform our modeling strategy. We incorporate these key features to capture green transition dynamics in small open economies. First, the economy consists of both brown and green intermediate goods producers. Brown producers are carbon intensive, while green producers rely on cleaner technologies and renewable energy. Following Minesso and Pagliari (2023), we assume that green producers face lower effective capital returns due to the higher costs associated with green investments in energy, infrastructure, and technology. Second, we impose a carbon tax on emissions generated by brown producers, calibrated to reflect observed policy practices. For example, Singapore's carbon tax reaches 25 Singapore dollars per ton of CO_2, while South Korea's implicit carbon pricing corresponds to approximately 6.46 euros per ton in 2023. Although carbon prices vary across economies, our analysis considers a range of tax levels and monetary policy regimes, allowing us to systematically evaluate the macroeconomic and environmental effects of coordinated carbon pricing and monetary policy in small open economies. Third, we embed the cap-and-trade policy into our framework, as this environmental regulation measure has been featured in many carbon credit transaction markets. Additionally, the cap-and-trade framework can provides us with a comparable and contending case of evaluating the economic performance and welfare implications between different environmental policies.

3 Theoretical Model

3.1 Model Overview

The model consists of three main blocks: the household sector, the production sector, and the government sector.

The household sector features a representative household that derives utility from consumption and disutility from labor supply. As the owner of domestic firms, the household chooses investment, which accumulates into capital subject to depreciation and investment adjustment costs. The household receives lump-sum transfers from the government and can trade one-period domestic bonds denominated in domestic currency as well as one-period foreign bonds in incomplete international financial markets. This setup limits international risk sharing, ensures stationarity of net foreign asset positions through an asset-elastic risk premium on foreign bond holdings, and is consistent with empirical evidence.

Several features distinguish the small open economy environment from a closed economy. Household consumption is a composite of domestically produced and imported final goods. Labor is immobile across borders, and capital accumulation depends solely on domestic investment. As owners of domestic firms, households receive monopolistic profits from retail firms.

The production sector is multi-layered and consists of intermediate goods producers, retailers, and final goods producers. Intermediate goods producers operate under perfect competition and produce using capital and labor via a Cobb–Douglas technology. Retailers source intermediate goods to produce differentiated varieties and set prices subject to Calvo-style nominal rigidities. Retail goods are then aggregated by final goods producers in perfectly competitive markets.

To model international trade while preserving tractability, trade occurs only at the final goods level. Domestic final goods producers aggregate sectoral outputs into a home-produced composite good and import a foreign-produced composite good from the rest of the world. These two components are combined into an Armington final good, which serves as the economy's final output. This approach follows Kolasa et al. (2025) and Justiniano and Preston (2010) and allows us to focus on environmental and monetary policy interactions without introducing excessive complexity.

The green transition is captured by introducing two parallel production sectors: a carbon-intensive brown sector and a clean green sector. Brown intermediate goods production generates carbon emissions, while green production does not. In each sector, monopolistically competitive retailers produce differentiated varieties, set prices under Calvo frictions, and supply sector-specific goods. Sectoral aggregators bundle varieties within each sector into composite brown and green goods. A domestic final goods producer combines these sectoral composites into a home-produced final good, which is subsequently combined with imported goods to form the Armington final good.

The government sector introduces monetary and environmental policies. Monetary policy is conducted by a central bank that sets the nominal interest rate according to a Taylor-type rule. Environmental policy is implemented through a carbon tax on emissions from brown production or, alternatively, through an emissions cap. Both instruments are treated as exogenously set policy rules, reflecting the practice that governments typically announce carbon prices or emissions targets in advance.

Following the International Monetary Fund's exchange rate classification, we distinguish monetary-exchange rate regimes along two dimensions: exchange rate flexibility and policy objectives. The model encompasses four representative regimes: a hard peg (fixed exchange rate), a managed float with exchange rate smoothing, a free float with strict inflation targeting, and a free float with dual targeting of inflation and output. These regimes capture the most prevalent policy frameworks observed among small open economies.

The model is closed with standard market-clearing conditions. Domestic output is allocated to consumption, investment, adjustment and abatement costs, and exports. The trade balance is mirrored by changes in net foreign asset positions in the international financial market.

3.2 Household

The representative household maximizes the following separable expected lifetime utility function:

$$U_t = \mathbb{E}t \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{h,t+s}^{1-\psi_C}}{1-\psi_C} - \phi_L \frac{L_{h,t+s}^{1+\psi_L}}{1+\psi_L} \right), \tag{1}$$

where β denotes the subjective discount factor. We set $\beta = 0.99$, which implies an annualized steady-state real interest rate of approximately 4%. The parameter ψ_C governs the curvature of utility with respect to consumption and corresponds to the coefficient of relative risk aversion (CRRA). Higher values of ψ_C imply stronger consumption smoothing across periods. We set $\psi_C = 1.5$, which lies within the standard range of values used in the literature, typically between 1 and 3. The parameter ψ_L is the inverse of the Frisch elasticity of labor supply and captures the responsiveness of labor supply to changes in wages. A higher ψ_L implies a less elastic labor supply. We set $\psi_L = 1$, a commonly used benchmark value. Finally, ϕ_L scales the disutility of labor and is calibrated to match a steady-state labor supply of one-third of available time in an economy without carbon taxation.

One parameter warrants additional clarification: ϕ_L. To match the steady-state labor supply, we calibrate ϕ_L to 6.9164 so that steady-state labor hours equal $\frac{1}{3}$ under the benchmark regime without carbon taxation. To ensure meaningful comparisons across policy experiments, we hold this calibrated value of ϕ_L fixed when evaluating alternative carbon tax rates. The same treatment is applied in the analysis of the cap-and-trade policy, allowing differences in outcomes to be attributed solely to policy changes rather than to shifts in labor preferences.

As we model a small open economy, total household consumption is defined as a CES composite of domestically produced and imported goods:

$$C_{h,t} = \left(\omega_C^{\frac{1}{\eta_C}} C_{d,t}^{\frac{\eta_C-1}{\eta_C}} + (1-\omega_C)^{\frac{1}{\eta_C}} C_{m,t}^{\frac{\eta_C-1}{\eta_C}} \right)^{\frac{\eta_C}{\eta_C-1}}, \tag{2}$$

where $C_{d,t}$ and $C_{m,t}$ denote aggregate consumption of domestically produced and imported goods, respectively. The parameter η_C governs the elasticity of substitution between domestic and imported consumption, while $\omega_C \in (0,1)$ captures home bias and thus reflects the degree of trade openness.

Following the two-sector (brown and green) structure of the E-DSGE model, aggregate domestic consumption is defined as a CES composite of brown and green goods:

$$C_{d,t} = \left(\omega_d^{\frac{1}{\sigma_d}} C_{h,b,t}^{\frac{\sigma_d - 1}{\sigma_d}} + (1 - \omega_d)^{\frac{1}{\sigma_d}} C_{h,g,t}^{\frac{\sigma_d - 1}{\sigma_d}} \right)^{\frac{\sigma_d}{\sigma_d - 1}}, \tag{3}$$

where $C_{h,b,t}$ and $C_{h,g,t}$ denote domestic consumption of brown and green goods, respectively. The parameter $\omega_d \in (0,1)$ represents the expenditure share on brown goods in steady state, while σ_d governs the elasticity of substitution between brown and green consumption.

Following Gali and Monacelli (2005) and Lozej et al. (2023), we derive the consumer price index (CPI), denoted by $P_{h,t}$, as the price index of the final consumption good that aggregates both domestically produced and imported goods. Similarly, $P_{d,t}$ denotes the aggregate price index of domestically produced final goods. These price indices are given by

$$P_{d,t} = \left(\omega_d P_{h,b,t}^{1-\sigma_d} + (1 - \omega_d) P_{h,g,t}^{1-\sigma_d} \right)^{\frac{1}{1-\sigma_d}}, \tag{4}$$

$$P_{h,t} = \left(\omega_C P_{d,t}^{1-\eta_C} + (1 - \omega_C) P_{m,t}^{1-\eta_C} \right)^{\frac{1}{1-\eta_C}}, \tag{5}$$

where $P_{h,b,t}$ and $P_{h,g,t}$ denote the prices of domestic brown and green goods, respectively, and $P_{m,t}$ is the price of the imported composite good expressed in domestic currency.

Cost minimization by households implies the following demand functions:

$$C_{d,t} = \omega_C \left(\frac{P_{d,t}}{P_{h,t}} \right)^{-\eta_C} C_{h,t}, \tag{6}$$

$$C_{m,t} = (1 - \omega_C) \left(\frac{P_{m,t}}{P_{h,t}} \right)^{-\eta_C} C_{h,t}, \tag{7}$$

$$C_{h,b,t} = \omega_d \left(\frac{P_{h,b,t}}{P_{d,t}} \right)^{-\sigma_d} C_{d,t}, \tag{8}$$

$$C_{h,g,t} = (1 - \omega_d) \left(\frac{P_{h,g,t}}{P_{d,t}} \right)^{-\sigma_d} C_{d,t}. \tag{9}$$

We assume producer-currency pricing for imported goods. As a result, the nominal exchange rate plays a key role in translating foreign producer prices into domestic-currency import prices.

The representative household maximizes expected utility subject to the following budget constraint:

$$\begin{aligned}
&P_{h,t}C_{h,t} + P_{h,t}I_{h,b,t} + P_{h,t}I_{h,g,t} + B_{h,t} + S_t B^*_{h,t} \\
&\leq \; W_{h,b,t}L_{h,b,t} + W_{h,g,t}L_{h,g,t} + R^K_{h,b,t}K_{h,b,t} + R^K_{h,g,t}K_{h,g,t} \\
&\quad + R_{h,t-1}B_{h,t-1} + S_t R^*_{t-1}\,\Gamma\big(B^*_{h,t-1}\big)\,B^*_{h,t-1} \\
&\quad - P_{h,t}\frac{\gamma_I}{2}\left(\frac{I_{h,b,t}}{K_{h,b,t}} - \delta\right)^2 K_{h,b,t} - P_{h,t}\frac{\gamma_I}{2}\left(\frac{I_{h,g,t}}{K_{h,g,t}} - \delta\right)^2 K_{h,g,t} \\
&\quad + \pi^{ret}_{h,b,t} + \pi^{ret}_{h,g,t} + T_{h,t}.
\end{aligned} \tag{10}$$

Consumption and investment expenditures, as well as investment adjustment costs in both sectors, are denominated in units of the final composite good, with price $P_{h,t}$. Labor income and capital income are sector specific, with wages $W_{h,b,t}$ and $W_{h,g,t}$ and rental rates $R^K_{h,b,t}$ and $R^K_{h,g,t}$. The household can trade one-period domestic and foreign bonds, where foreign bond returns are adjusted by a risk-premium function $\Gamma(\cdot)$. Retail profits from both brown and green sectors, $\pi^{ret}_{h,b,t}$ and $\pi^{ret}_{h,g,t}$, are rebated lump-sum to the household, along with government transfers $T_{h,t}$.

Another element that warrants further discussion is household bond holding. The domestic household can purchase one-period domestic bonds, denoted by $B_{h,t}$, and one-period foreign bonds, denoted by $B^*_{h,t}$, which are denominated in foreign currency. The nominal exchange rate at time t is denoted by S_t. To capture imperfect access to international financial markets and to ensure stationarity of net foreign asset positions, we introduce an asset-elastic risk premium, a standard assumption in small open economy models with incomplete international financial markets. The risk-premium function, $\Gamma(\cdot)$, is specified as

$$\Gamma(B^*_{h,t}) = exp\left[\psi_b\left(\frac{S_t B^*_{h,t}}{P_{h,t}Y_{h,t}} - \bar{b}^*\right)\right], \tag{11}$$

where ψ_b governs the sensitivity of the risk premium to deviations of the net foreign asset position from its steady-state level. The term $\frac{S_t B^*_{h,t}}{P_{h,t}Y_{h,t}}$ represents foreign asset holdings scaled by aggregate output, and $\bar{b}^*$ denotes the steady-state ratio of net foreign assets to output.

Capital accumulation in the brown and green sectors follows standard laws of motion:

$$K_{h,b,t+1} = I_{h,b,t} + (1-\delta)K_{h,b,t}, \tag{12}$$

$$K_{h,g,t+1} = I_{h,g,t} + (1-\delta)K_{h,g,t}, \tag{13}$$

where δ denotes the depreciation rate, which is assumed to be identical across sectors.

Aggregate labor supply, $L_{h,t}$, is modeled as a CES composite of labor supplied to the brown and green sectors:

$$L_{h,t} = \left(\omega_L L_{h,b,t}^{\rho_L} + (1-\omega_L) L_{h,g,t}^{\rho_L}\right)^{\frac{1}{\rho_L}}, \tag{14}$$

where $\rho_L \equiv \frac{v_L - 1}{v_L}$ and v_L denotes the elasticity of substitution between labor hours employed in the brown and green sectors. This parameter governs the ease with which households can reallocate labor across sectors. The parameter ω_L pins down the steady-state share of labor allocated to the brown sector. This labor aggregation implies imperfect labor mobility across sectors, allowing for sector-specific wages and capturing frictions arising from skill specificity and adjustment costs during the green transition.

We characterize the household's optimization problem subject to the budget constraint and capital accumulation equations. The resulting first-order conditions are provided in the Appendix.

3.3 Intermediate Goods Producer

We now describe the production side of the model. Intermediate goods producers are perfectly competitive and employ a Cobb–Douglas production function to produce brown and green intermediate goods.

$$X_{h,b,t} = (1 - V_{h,t}) A_t L_{h,b,t}^{1-\alpha} K_{h,b,t}^{\alpha} \tag{15}$$

$$X_{h,g,t} = (1 - V_{h,t}) A_t L_{h,g,t}^{1-\alpha} (\psi_g K_{h,g,t})^{\alpha} \tag{16}$$

The production functions of brown and green intermediate goods producers share the same functional form, with one key distinction. Following Minesso and Pagliari (2023), we introduce a parameter ψ_g to capture differences in capital productivity across sectors. Specifically, brown producers are assumed to exhibit higher marginal productivity of capital, reflecting more mature, standardized, and reliable technologies, while green producers face lower effective capital returns. Accordingly, we set $\psi_g \in (0, 1)$.

We incorporate this productivity differential in a way that preserves constant returns to scale and the zero-profit condition under perfect competition. In particular, ψ_g enters as a sector-specific productivity shifter that reduces the effective capital services provided per unit of installed green capital. Thus, although green firms use the same production technology as brown firms, each unit of green capital yields fewer effective services. This modeling choice maintains tractability while capturing the lower capital efficiency typically associated with green technologies during the transition phase.

The parameter α denotes the capital share in production and is calibrated to 0.37. In addition, we incorporate an output loss function, $\mathcal{L}(\cdot)$, which maps total emissions M_t into production losses. This approach follows the environmental DSGE literature and is closely related to the damage function used in Nordhaus's DICE framework. The loss function is specified as

$$\mathcal{L}(M_t) = d_0 + d_1 M_t + d_2 M_t^2. \tag{17}$$

To capture the open-economy nature of environmental damages, we distinguish between two types of emission-related losses: $\mathcal{L}_w$ and $\mathcal{L}_d$. The term $\mathcal{L}_w$ represents output losses arising from emissions in the rest of the world, while $\mathcal{L}_d$ captures losses generated by domestic emissions. This distinction reflects the fact that a small open economy is negligible relative to the global economy but remains materially affected by its own domestic emissions. Separating these two channels allows us to isolate the role of domestic environmental policy while accounting for exogenous global emission damages.

Output losses arising from emissions in the rest of the world are given by

$$\mathcal{L}_w(M_t^*) = \omega_h \left(d_0 + d_1 M_t^* + d_2 (M_t^*)^2 \right), \tag{18}$$

where ω_h captures the extent to which global emissions are internalized by the small open economy, and M_t^* denotes total foreign emissions. The parameters d_1 and d_2 governing the linear and quadratic components of emission damages are discussed later in this subsection.

Domestic emission-related output losses are specified as

$$\mathcal{L}_d(M_{h,t}) = d_0 + d_1 M_{h,t} + d_2 M_{h,t}^2, \tag{19}$$

where $M_{h,t}$ denotes the domestic emission stock at time t. The evolution of the domestic emission stock follows

$$M_{h,t} = \rho_m M_{h,t-1} + (1 - \kappa_{h,b,t})\, \zeta(X_{h,b,t})\,, \tag{20}$$

with $\rho_m \in (0,1)$ governing the persistence of emissions over time. Current emissions depend on polluting brown-sector output $X_{h,b,t}$, scaled by the emissions intensity ζ and reduced by the abatement effort $\kappa_{h,b,t}$.

Abatement effort entails a resource cost, captured by the abatement cost function

$$\Phi_{h,t} = \varepsilon_1 \kappa_{h,b,t}^{\varepsilon_2} X_{h,b,t}, \tag{21}$$

where $\varepsilon_1 > 0$ and $\varepsilon_2 > 1$ govern the level and curvature of abatement costs.

The domestic emission stock $M_{h,t}$ thus reflects both the accumulation of past emissions and the flow of newly generated emissions net of abatement. This formulation allows emissions to persist over time while providing a clear channel through which environmental policy affects production costs and output.

The parameter ρ_m denotes the persistence of emissions and governs the extent to which the current emission stock depends on its lagged value. We calibrate ρ_m to 0.95, implying that emissions are highly persistent over time. The parameter ζ measures the emissions intensity of brown production, capturing the amount of carbon emissions generated per unit of polluting output. Following the literature, we set $\zeta = 0.45$, indicating that 45% of each additional unit of brown output is converted into carbon emissions.

The parameters d_0, d_1, and d_2 in the loss function govern how emissions translate into output losses. Specifically, d_0 and d_1 determine the level and linear component of emission-related damages, while d_2 controls the curvature of the damage function and thus the rate at which marginal damages increase with emissions. We calibrate these parameters following Heutel (2012) and Annicchiarico and Di Dio (2015), ensuring consistency with the environmental DSGE literature.

One aspect that warrants further clarification is the relationship between output losses and the stock of emissions. Following the environmental economics literature, we assume that output adjusts contemporaneously to emissions. Specifically, current-period emission flows are assumed to enter the atmosphere immediately, adding to the existing stock of accumulated emissions. As a result, the total emission stock in the atmosphere directly affects output in the same period.

We also distinguish between domestic emission stock, $M_{h,t}$, and emissions originating in the rest of the world, M_t^*. This distinction is essential for our small open economy setting. Since the home economy is assumed to be infinitesimally small relative to the global economy, its emissions have a negligible impact on world emission levels. Aggregating domestic emissions directly into global emissions would therefore obscure the role of domestic environmental policies. By separating domestic and foreign emission stocks, we are able to isolate the effects of home environmental policies on domestic output while treating global emissions as an exogenous source of environmental damage.

Total output losses in the home economy are defined as the sum of losses arising from global and domestic emissions:

$$V_{h,t} = \mathcal{L}_w(M_t^*) + \mathcal{L}_d(M_{h,t}). \tag{22}$$

Equation (21) (to be renumbered) specifies the cost associated with emission abatement. The variable $\kappa_{h,b,t}$ denotes the abatement effort undertaken by brown intermediate goods producers to reduce carbon emissions. The parameter ε_1 scales the level of abatement costs, while ε_2 governs the curvature of the cost function and thus the rate at which marginal abatement costs increase with effort. Following the environmental economics literature, we calibrate ε_1 to 1.17 and ε_2 to 2.8.

It is useful to note that, in equilibrium, the marginal cost of abatement is equal to the carbon tax levied per unit of emissions. Formally, this condition can be expressed as

$$\zeta \cdot \frac{P_{z,t}}{P_{h,t}} = \varepsilon_1 \varepsilon_2 \kappa_{h,b,t}^{\varepsilon_2 - 1}, \tag{23}$$

where ζ denotes the emissions intensity of production, $P_{z,t}$ is the nominal carbon price per ton of CO2, and $P_{h,t}$ is the price of the final composite good.

This condition equates the marginal benefit of abatement, measured by the reduction in carbon tax payments from lowering emissions, to the marginal cost of abatement effort. As abatement increases, marginal costs rise at a rate governed by ε_2, while higher carbon prices provide stronger incentives for firms to undertake abatement.

Intermediate goods producers in both the brown and green sectors operate under perfect competition. Their period profits are given by

$$\pi_{h,b,t} = P_{x,b,t} X_{h,b,t} - R_{h,b,t}^K K_{h,b,t} - W_{h,b,t} L_{h,b,t}, \tag{24}$$

$$\pi_{h,g,t} = P_{x,g,t}X_{h,g,t} - R^K_{h,g,t}K_{h,g,t} - W_{h,g,t}L_{h,g,t}. \tag{25}$$

Firms in both sectors choose capital and labor to maximize profits. Under perfect competition and constant returns to scale, profit maximization implies zero profits in equilibrium.

As indicated above, intermediate goods prices, $P_{x,b,t}$ and $P_{x,g,t}$, are taken as given by brown and green intermediate goods producers. Under perfect competition, these prices equal marginal production costs. For brown producers, marginal costs additionally reflect environmental costs associated with emissions and abatement. To link intermediate goods prices to marginal costs, we derive the marginal cost expressions for intermediate goods producers. The marginal cost of brown intermediate goods, $MC_{h,b,t}$, consists of three components: the marginal cost of production inputs, denoted by $Q_{h,b,t}$; the marginal abatement cost per unit of output, given by $P_{h,t}\varepsilon_1\kappa^{\varepsilon_2}_{h,b,t}$; and the carbon tax paid on residual emissions, $(1-\kappa_{h,b,t})\zeta P_{z,t}$, generated by an additional unit of brown output. Formally,

$$MC_{h,b,t} = Q_{h,b,t} + P_{h,t}\varepsilon_1\kappa^{\varepsilon_2}_{h,b,t} + (1-\kappa_{h,b,t})\zeta P_{z,t}. \tag{26}$$

We derive production marginal costs by solving the cost-minimization problem of intermediate goods producers. The marginal cost component associated with production inputs for brown intermediate goods, denoted by $Q_{h,b,t}$, is given by

$$Q_{h,b,t} = \frac{W^{1-\alpha}_{h,b,t}\left(R^K_{h,b,t}\right)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}A_t\left(1-V_{h,t}\right)}, \tag{27}$$

where A_t denotes total factor productivity and $V_{h,t}$ captures output losses due to emission-related damages.

Under perfect competition, the zero-profit condition implies that the price of brown intermediate goods equals their marginal cost. Accordingly,

$$P_{x,b,t} = MC_{h,b,t} = Q_{h,b,t} + P_{h,t}\varepsilon_1\kappa^{\varepsilon_2}_{h,b,t} + (1-\kappa_{h,b,t})\zeta P_{z,t}. \tag{28}$$

The real marginal cost faced by brown-sector retailers is therefore given by

$$mc_{h,b,t} = \frac{P_{x,b,t}}{P_{h,b,t}}, \tag{29}$$

where $P_{h,b,t}$ denotes the price index of brown-sector retail goods.

In a similar manner, we derive the marginal cost, intermediate goods price, and real marginal cost for green-sector firms. Since green intermediate goods production does not generate emissions, the total marginal cost of green intermediate goods producers is given solely by production costs:

$$MC_{h,g,t} = Q_{h,g,t}. \tag{30}$$

The marginal cost component associated with production inputs is

$$Q_{h,g,t} = \frac{W_{h,g,t}^{1-\alpha} \left(R_{h,g,t}^{K}\right)^{\alpha}}{(1 - V_{h,t}) A_t \psi_g^{\alpha} (1-\alpha)^{1-\alpha} \alpha^{\alpha}}, \tag{31}$$

where $\psi_g \in (0, 1)$ captures the lower effective productivity of green capital relative to brown capital.

Under perfect competition, the zero-profit condition implies that the price of green intermediate goods equals marginal cost:

$$P_{x,g,t} = MC_{h,g,t}. \tag{32}$$

Finally, the real marginal cost faced by green-sector retailers is defined as

$$mc_{h,g,t} = \frac{P_{x,g,t}}{P_{h,g,t}}, \tag{33}$$

where $P_{h,g,t}$ denotes the price index of green-sector retail goods.

3.4 Retailers

We next turn to the pricing decisions of retailers. Retail firms assemble intermediate goods into differentiated varieties and set prices subject to Calvo-type nominal rigidities.

In each period, only a fraction $1 - \omega$ of retailers are allowed to reoptimize their prices, while the remaining fraction ω keep their prices unchanged from the previous period. This pricing structure applies symmetrically to both brown and green retail sectors. For firms that cannot adjust prices, the pricing rules are given by

$$P_{h,b,t}(i) = P_{h,b,t-1}(i), \tag{34}$$

$$P_{h,g,t}(j) = P_{h,g,t-1}(j), \tag{35}$$

where i and j index individual varieties in the brown and green retail sectors, respectively.

To determine the optimal reset prices, we consider the profit maximization problem faced by reoptimizing retailers in the brown and green sectors. A retailer that is allowed to reset its price in period t chooses a constant price to maximize the expected discounted stream of profits over the periods during which the price remains in effect.

For a brown-sector retailer, the problem is given by

$$\max_{P^{\text{opt}}_{h,b,t}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \frac{\lambda_{t+s}}{\lambda_t} y_{h,b,t+s|t} \left[P^{\text{opt}}_{h,b,t} - P_{h,b,t+s} mc_{h,b,t+s} \right], \tag{36}$$

and analogously, for a green-sector retailer,

$$\max_{P^{\text{opt}}_{h,g,t}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \frac{\lambda_{t+s}}{\lambda_t} y_{h,g,t+s|t} \left[P^{\text{opt}}_{h,g,t} - P_{h,g,t+s} mc_{h,g,t+s} \right]. \tag{37}$$

Here, λ_t denotes the household's marginal utility of consumption, and $y_{h,b,t+s|t}$ and $y_{h,g,t+s|t}$ represent demand for a retailer's variety in periods $t+s$ conditional on the price being set in period t. The real marginal costs faced by retailers are defined as

$$mc_{h,b,t} = \frac{P_{x,b,t}}{P_{h,b,t}}, \tag{38}$$

$$mc_{h,g,t} = \frac{P_{x,g,t}}{P_{h,g,t}}. \tag{39}$$

Demand for an individual retail variety depends on its relative price and aggregate demand in the corresponding sector. Specifically, demand for variety i in the brown retail sector and variety j in the green retail sector is given by

$$y_{h,b,t+s|t}(i) = \left(\frac{P^{\text{opt}}_{h,b,t}}{P_{h,b,t+s}} \right)^{-\rho} Z_{h,b,t+s}, \tag{40}$$

$$y_{h,g,t+s|t}(j) = \left(\frac{P^{\text{opt}}_{h,g,t}}{P_{h,g,t+s}} \right)^{-\rho} Z_{h,g,t+s}, \tag{41}$$

where ρ denotes the elasticity of substitution across varieties within each retail sector. The variables $Z_{h,b,t+s}$ and $Z_{h,g,t+s}$ represent aggregate demand for brown and green retail goods in period $t+s$, respectively.

These aggregate sectoral goods are constructed by sector-specific aggregators that bundle all differentiated retail varieties into composite brown and green goods. This structure ensures that individual retailers face downward-sloping demand curves and take aggregate sectoral demand as given when setting prices.

Formally, sectoral aggregators bundle all differentiated retail varieties into composite sectoral goods. The brown-sector aggregator combines individual brown retail varieties into a composite brown good,

$$Z_{h,b,t} = \left(\int_0^1 y_{h,b,t}(i)^{\frac{\rho-1}{\rho}} di \right)^{\frac{\rho}{\rho-1}}, \tag{42}$$

while the green-sector aggregator constructs a composite green good,

$$Z_{h,g,t} = \left(\int_0^1 y_{h,g,t}(j)^{\frac{\rho-1}{\rho}} dj \right)^{\frac{\rho}{\rho-1}}. \tag{43}$$

The parameter ρ denotes the elasticity of substitution across varieties and is assumed to be identical in the brown and green sectors. By symmetry, all retailers that are able to reoptimize prices in period t choose the same optimal price, denoted by $P^{\text{opt}}_{h,b,t}$ or $P^{\text{opt}}_{h,g,t}$. As a result, demand is identical across all reoptimizing varieties, allowing us to suppress the variety index without loss of generality in subsequent expressions.

Taking into account that all reoptimizing retailers within each sector choose the same price, we solve for the optimal reset prices $P^{\text{opt}}_{h,b,t}$ and $P^{\text{opt}}_{h,g,t}$. After standard algebraic manipulations, the optimal prices are given by

$$P^{\text{opt}}_{h,b,t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \frac{\lambda_{t+s}}{\lambda_t} y_{h,b,t+s|t} P^{\rho}_{h,b,t+s} mc_{h,b,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \frac{\lambda_{t+s}}{\lambda_t} y_{h,b,t+s|t} P^{\rho-1}_{h,b,t+s}} \cdot \frac{\rho}{\rho-1}, \tag{44}$$

$$P^{\text{opt}}_{h,g,t} = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \frac{\lambda_{t+s}}{\lambda_t} y_{h,g,t+s|t} P^{\rho}_{h,g,t+s} mc_{h,g,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \frac{\lambda_{t+s}}{\lambda_t} y_{h,g,t+s|t} P^{\rho-1}_{h,g,t+s}} \cdot \frac{\rho}{\rho-1}. \tag{45}$$

These expressions take the familiar form of a constant markup, $\rho/(\rho-1)$, over a discounted average of expected future marginal costs, reflecting the Calvo pricing friction in each sector.

The sectoral price indices for domestic retail goods follow standard Dixit–Stiglitz aggregators. For the brown and green retail sectors, respectively, the price indices are given by

$$P_{h,b,t} = \left[(1-\omega)\left(P^{\text{opt}}_{h,b,t}\right)^{1-\rho} + \omega\left(P_{h,b,t-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{46}$$

$$P_{h,g,t} = \left[(1-\omega)\left(P^{\text{opt}}_{h,g,t}\right)^{1-\rho} + \omega\left(P_{h,g,t-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{47}$$

where $\rho > 1$ denotes the elasticity of substitution across differentiated retail varieties. A higher value of ρ implies greater substitutability across varieties and a lower desired markup. We calibrate ρ to 6, a standard value in the New Keynesian literature.

3.5 Final Goods Producers

Following standard practice in the macroeconomics literature, we introduce a representative final goods producer. Production at this stage involves two layers of aggregation. In the first layer, a domestically produced composite good is manufactured by combining the brown and green sectoral retail goods. In the second layer, the final goods producer imports a foreign composite good, constructed analogously from foreign brown and green sectoral retail goods, and combines it with the domestic composite good to produce an Armington-style final good. The domestically produced composite good is also exported to the rest of the world. We describe each aggregation stage in detail below.

The domestic final goods producer first combines the brown and green sectoral retail goods to produce a domestically composite good. The domestically produced composite good, $Y_{h,t}$, is formed using a CES aggregation of brown and green sectoral retail goods:

$$Y_{h,t} = \left[\omega_d Z_{h,b,t}^{\frac{\eta-1}{\eta}} + (1-\omega_d) Z_{h,g,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{48}$$

where ω_d denotes the steady-state expenditure share on the brown sectoral good and is calibrated following Minesso and Pagliari (2023). The parameter η represents the elasticity of substitution between brown and green sectoral goods and governs the ease with which production shifts from carbon-intensive to green inputs.

The final goods producer chooses inputs of brown and green sectoral retail goods, $Z_{h,b,t}$ and $Z_{h,g,t}$, to maximize profits:

$$P_{d,t}Y_{h,t} - P_{h,b,t}Z_{h,b,t} - P_{h,g,t}Z_{h,g,t}, \tag{49}$$

where $P_{d,t}$ denotes the price of the domestically produced composite good. Cost minimization implies the following input demand functions:

$$Z_{h,b,t} = \omega_d \left(\frac{P_{h,b,t}}{P_{d,t}} \right)^{-\eta} Y_{h,t}, \tag{50}$$

$$Z_{h,g,t} = (1 - \omega_d) \left(\frac{P_{h,g,t}}{P_{d,t}} \right)^{-\eta} Y_{h,t}. \tag{51}$$

The price index for the domestically produced final good is given by

$$P_{d,t} = \left[\omega_d P_{h,b,t}^{1-\eta} + (1 - \omega_d) P_{h,g,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{52}$$

which ensures the zero-profit condition under perfect competition in the final goods market.

We now turn to the second layer of aggregation, which introduces international trade. Building on the domestic composite good defined in the first layer, we specify an Armington aggregation for the final good. The final aggregated good, Y_t, is given by

$$Y_t = \left[\omega_C^{\frac{1}{\eta_y}} Y_{h,t}^{\frac{\eta_y - 1}{\eta_y}} + (1 - \omega_C)^{\frac{1}{\eta_y}} \left(Y_t^* \right)^{\frac{\eta_y - 1}{\eta_y}} \right]^{\frac{\eta_y}{\eta_y - 1}}, \tag{53}$$

where Y_t^* denotes the foreign-produced composite good, constructed analogously as an aggregation of foreign brown and green sectoral retail goods. The parameter $\omega_C \in (0, 1)$ captures home bias and thus reflects the degree of trade openness, while η_y governs the elasticity of substitution between domestically produced and foreign composite goods.

Although η_y and η_C both characterize substitution between home and foreign goods, they enter different parts of the model: η_y applies to final goods production, whereas η_C applies to household consumption. For simplicity and without loss of generality, we set $\eta_y = \eta_C$, thereby imposing a common elasticity of substitution between domestic and foreign goods across production and consumption.

The price index of the final aggregated good, which also serves as the consumer price index (CPI), is defined as

$$P_{h,t} = \left[\omega_C P_{d,t}^{1-\eta_C} + (1-\omega_C) P_{m,t}^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}, \tag{54}$$

where $P_{d,t}$ denotes the price of the domestically produced composite good and $P_{m,t}$ is the price of the imported composite good. Aggregate inflation is then defined in the standard way as

$$\Pi_t = \frac{P_{h,t}}{P_{h,t-1}}. \tag{55}$$

Cost minimization by the final goods producer implies the following demand functions for domestic and foreign composite goods:

$$Y_{h,t} = \omega_C \left(\frac{P_{d,t}}{P_{h,t}}\right)^{-\eta_y} Y_t, \tag{56}$$

$$Y_t^* = (1-\omega_C) \left(\frac{P_{m,t}}{P_{h,t}}\right)^{-\eta_y} Y_t. \tag{57}$$

These expressions characterize the allocation between domestically produced and imported goods as a function of relative prices and the elasticity of substitution.

Since we assume producer-currency pricing and full exchange rate pass-through, the domestic-currency price of imported goods is given by

$$P_{m,t} = S_t P_t^*, \tag{58}$$

where S_t denotes the nominal exchange rate and P_t^* is the foreign aggregate price level.

To characterize the evolution of P_t^*, we specify exogenous processes for foreign inflation and the world interest rate. Foreign inflation, Π_t^*, follows an AR(1) process:

$$\log(\Pi_t^*) = \rho_\pi^* \log(\Pi_{t-1}^*) + (1-\rho_\pi^*) \log(\bar{\Pi}^*) + \epsilon_{\pi,t}^*, \tag{59}$$

which implies the following law of motion for the foreign price level:

$$P_t^* = \Pi_t^* P_{t-1}^*. \tag{60}$$

Similarly, the world nominal interest rate, R_t^*, evolves according to

$$\log(R_t^*) = \rho_r^* \log(R_{t-1}^*) + (1-\rho_r^*) \log(\bar{R}^*) + \epsilon_{r,t}^*, \tag{61}$$

where $\bar{\Pi}^*$ and $\bar{R}^*$ denote the steady-state levels of foreign inflation and the world interest rate, respectively, and $\epsilon^*_{\pi,t}$ and $\epsilon^*_{r,t}$ are i.i.d. shocks.

We also account for price dispersion by aggregating individual retail varieties over the unit interval. The total physical output produced by brown and green retail firms is given by

$$y_{h,b,t} = \int_0^1 y_{h,b,t}(i)di, \tag{62}$$

$$y_{h,g,t} = \int_0^1 y_{h,g,t}(j)dj. \tag{63}$$

These expressions represent the raw sum of physical units produced across all brown and green retail firms, respectively. In contrast, $Z_{h,b,t}$ and $Z_{h,g,t}$ denote the corresponding effective aggregate composite goods, which account for price dispersion induced by nominal rigidities. As a result, the two measures generally differ whenever prices are not fully flexible.

Within each sector, demand for an individual retail variety depends on its relative price and aggregate sectoral demand. Specifically, demand for brown variety i and green variety j is given by

$$y_{h,b,t}(i) = \left(\frac{P_{h,b,t}(i)}{P_{h,b,t}}\right)^{-\rho} Z_{h,b,t}, \tag{64}$$

$$y_{h,g,t}(j) = \left(\frac{P_{h,g,t}(j)}{P_{h,g,t}}\right)^{-\rho} Z_{h,g,t}. \tag{65}$$

Integrating over all varieties yields total physical output in each sector:

$$y_{h,b,t} = \int_0^1 \left(\frac{P_{h,b,t}(i)}{P_{h,b,t}}\right)^{-\rho} Z_{h,b,t}di, \tag{66}$$

$$y_{h,g,t} = \int_0^1 \left(\frac{P_{h,g,t}(j)}{P_{h,g,t}}\right)^{-\rho} Z_{h,g,t}dj. \tag{67}$$

Rearranging terms, we obtain

$$y_{h,b,t} = Z_{h,b,t} \int_0^1 \left(\frac{P_{h,b,t}(i)}{P_{h,b,t}}\right)^{-\rho} di, \tag{68}$$

$$y_{h,g,t} = Z_{h,g,t} \int_0^1 \left(\frac{P_{h,g,t}(j)}{P_{h,g,t}}\right)^{-\rho} dj. \tag{69}$$

We define sector-specific price dispersion terms as

$$D_{h,b,t} \equiv \int_0^1 \left(\frac{P_{h,b,t}(i)}{P_{h,b,t}} \right)^{-\rho} di, \tag{70}$$

$$D_{h,g,t} \equiv \int_0^1 \left(\frac{P_{h,g,t}(j)}{P_{h,g,t}} \right)^{-\rho} dj. \tag{71}$$

These definitions imply a direct relationship between effective composite output and total physical production:

$$Z_{h,b,t} = \frac{y_{h,b,t}}{D_{h,b,t}}, \tag{72}$$

$$Z_{h,g,t} = \frac{y_{h,g,t}}{D_{h,g,t}}. \tag{73}$$

Price dispersion thus captures the wedge between effective sectoral output used in final goods production and the sum of physical units produced across firms, arising from nominal price rigidities.

Under Calvo pricing, sectoral price dispersion evolves endogenously over time. The price dispersion terms for the brown and green retail sectors satisfy the following recursive equations:

$$D_{h,b,t} = (1-\omega) \left(\frac{P^{\text{opt}}_{h,b,t}}{P_{h,b,t}} \right)^{-\rho} + \omega \left(\Pi_{h,b,t}\right)^{\rho} D_{h,b,t-1}, \tag{74}$$

$$D_{h,g,t} = (1-\omega) \left(\frac{P^{\text{opt}}_{h,g,t}}{P_{h,g,t}} \right)^{-\rho} + \omega \left(\Pi_{h,g,t}\right)^{\rho} D_{h,g,t-1}, \tag{75}$$

where sectoral inflation rates are defined as $\Pi_{h,b,t} \equiv P_{h,b,t}/P_{h,b,t-1}$ and $\Pi_{h,g,t} \equiv P_{h,g,t}/P_{h,g,t-1}$.

We conclude this subsection by deriving the nominal profits of retail firms, which are rebated lump-sum to households and thus enter the household budget constraint. Retail profits in the brown and green sectors are given by

$$\pi^{\text{ret}}_{h,b,t} = \int_0^1 \left(P_{h,b,t}(i) - P_{x,b,t}\right) y_{h,b,t}(i) di = P_{h,b,t} Z_{h,b,t} - P_{x,b,t} D_{h,b,t} Z_{h,b,t}, \tag{76}$$

$$\pi^{\text{ret}}_{h,g,t} = \int_0^1 \left(P_{h,g,t}(j) - P_{x,g,t}\right) y_{h,g,t}(j) dj = P_{h,g,t} Z_{h,g,t} - P_{x,g,t} D_{h,g,t} Z_{h,g,t}. \tag{77}$$

These expressions highlight how nominal rigidities generate price dispersion, which in turn affects aggregate profits and resource allocation across sectors.

3.6 International Risk Sharing

In modeling a small open economy, we follow the standard approach in the literature by characterizing international financial linkages through the uncovered interest parity (UIP) condition under incomplete international financial markets. To ensure stationarity of net foreign asset positions, we introduce a debt-elastic risk premium.

With this specification, the UIP condition is given by

$$R_{h,t} = R_t^* \Gamma(B_{h,t}^*) \mathbb{E}t \left(\frac{S_{t+1}}{S_t} \right), \tag{78}$$

where $R_{h,t}$ denotes the gross nominal domestic interest rate, R_t^* is the gross world interest rate, and S_t is the nominal exchange rate. The risk premium function is specified as

$$\Gamma(B_{h,t}^*) = exp \left[\psi_b \left(\frac{S_t B_{h,t}^*}{P_{h,t} Y_{h,t}} - \bar{b}^* \right) \right], \tag{79}$$

where ψ_b governs the sensitivity of the risk premium to deviations of net foreign asset positions from their steady-state level, and $\bar{b}^*$ denotes the steady-state ratio of net foreign assets to output. This formulation captures imperfect international risk sharing while preserving a well-defined steady state.

3.7 Government Agencies

Before defining the equilibrium, we describe the policy-setting rules of the domestic fiscal authority and the central bank. The fiscal authority is responsible for taxation, while the monetary authority conducts interest rate policy following a Taylor-type rule. To keep the framework tractable, we abstract from several additional government activities. First, we do not consider government purchases or distortionary taxes other than the carbon tax. Incorporating government spending and alternative tax instruments is left for future research. Second, the central bank is assumed to operate solely through the policy interest rate and does not engage in other financial or macroprudential policies. These simplifications allow us to focus cleanly on the interaction between monetary policy and environmental regulation, particularly in response to interest rate shocks.

The fiscal authority imposes a carbon tax on emissions generated by domestic firms. The tax is levied at a constant rate per ton of carbon dioxide emitted and applies linearly to all residual emissions. While this specification is stylized, it captures the key features of carbon taxation adopted in practice. Specifically, for each unit of domestic carbon emissions in period t, total carbon tax revenue is given by

$$RE_{h,t} = P_{z,t}\,(1-\kappa_{h,b,t})\,\zeta(X_{h,b,t})\,, \tag{80}$$

where $P_{z,t}$ denotes the nominal carbon price per ton of CO_2, $\kappa_{h,b,t}$ is the abatement effort of brown-sector firms, and ζ maps polluting output into emissions. This formulation ensures that carbon tax revenues depend directly on firms' production decisions and abatement choices, providing a clear channel through which environmental policy affects the economy.

We assume that all carbon tax revenues are rebated lump-sum to households, consistent with the household budget constraint introduced earlier. The lump-sum transfer is therefore defined as

$$T_{h,t} = RE_{h,t}. \tag{81}$$

We assume that the central bank sets the policy interest rate according to a generalized Taylor rule. In a small open economy, the policy rate may respond not only to inflation and output, but also to exchange rate movements. The rule is specified as

$$\begin{aligned}\log(R_{h,t}) = {} & \rho_r \log\left(\frac{R_{h,t-1}}{R_{ss}}\right) \\ & + (1-\rho_r)\left[\phi_\pi \log\left(\frac{\Pi_{h,t}}{\Pi_{ss}}\right) + \phi_y \log\left(\frac{Y_{h,t}}{Y_{ss}}\right) + \phi_e \log\left(\frac{S_t}{S_{ss}}\right)\right] \\ & + \epsilon_t^r,\end{aligned} \tag{82}$$

where R_{ss}, Π_{ss}, Y_{ss}, and S_{ss} denote the steady-state levels of the policy rate, inflation, output, and the nominal exchange rate, respectively. The parameter ρ_r captures interest rate smoothing, while ϕ_π, ϕ_y, and ϕ_e govern the policy response to inflation, output, and exchange rate deviations. The term ϵ_t^r is an exogenous monetary policy shock.

Different monetary and exchange rate regimes are represented by alternative restrictions on (82).

Fixed exchange rate regime. Under a credible peg, the nominal exchange rate is constant,

$$S_t = \bar{S}, \tag{83}$$

so the uncovered interest parity condition reduces to

$$R_{h,t} = R_t^* \Gamma\big(B_{h,t}^*\big) . \tag{84}$$

Hence, the domestic interest rate is pinned down by the world interest rate and the country risk premium, implying little independent monetary policy.

Managed floating regime. Under managed floating, the central bank retains monetary policy autonomy but places explicit weight on exchange rate stabilization. We set

$$\rho_r = 0.74, \qquad \phi_\pi = 0, \qquad \phi_y = 0, \qquad \phi_e = 0.5.$$

Thus, policy mainly responds to exchange rate deviations, with inflation and output stabilization playing no direct role.

Freely floating regimes. Under a freely floating exchange rate, the exchange rate does not enter the policy rule, so $\phi_e = 0$. We consider two cases. First, under strict inflation targeting, the policy parameters are

$$\rho_r = 0.74, \qquad \phi_\pi = 1.5, \qquad \phi_y = 0.1.$$

This specification places primary weight on inflation stabilization, with only a limited response to output fluctuations. Second, under a dual-target regime, we set

$$\rho_r = 0.74, \qquad \phi_\pi = 1.5, \qquad \phi_y = 0.5.$$

Relative to strict inflation targeting, this regime assigns a stronger role to output stabilization alongside inflation control.

3.8 Market-clearing Conditions and General Equilibrium

We summarize the market-clearing conditions of the general equilibrium by defining total home output, denoted by $N_{h,t}$, as

$$N_{h,t} = Y_t + NX_{h,t}, \tag{85}$$

where $NX_{h,t}$ denotes net exports. This identity states that total domestic output equals the sum of domestic absorption and net exports.

In a closed economy, domestic absorption equals final output. In a small open economy, however, final output is allocated between domestic absorption and net exports. The resource constraint is therefore given by

$$\begin{aligned} Y_t = {} & C_{h,t} + I_{h,b,t} + I_{h,g,t} + \frac{\gamma_I}{2}\left(\frac{I_{h,b,t}}{K_{h,b,t}} - \delta\right)^2 K_{h,b,t} + \frac{\gamma_I}{2}\left(\frac{I_{h,g,t}}{K_{h,g,t}} - \delta\right)^2 K_{h,g,t} \\ & + \varepsilon_1 \, \kappa_{h,b,t}^{\varepsilon_2} X_{h,b,t}, \end{aligned} \tag{86}$$

where output is absorbed by consumption, investment in brown and green capital, investment adjustment costs, and abatement costs incurred by brown-sector firms. Net exports are defined as the difference between exports and imports:

$$NX_{h,t} = \mathcal{E}_{h,t} - \mathcal{I}_{h,t}. \tag{87}$$

Together with the net export identity introduced earlier, these conditions close the goods market in the small open economy.

The variables $\mathcal{E}_{h,t}$ and $\mathcal{I}_{h,t}$ denote exports of domestically produced goods and imports of foreign-produced goods, respectively. Imports are given by

$$\mathcal{I}_{h,t} = Y_{m,t}, \tag{88}$$

where $Y_{m,t}$ denotes real imports of the foreign composite good. Exports are defined as

$$\mathcal{E}_{h,t} = \omega_e \left(\frac{P^*_{e,t}}{P^*_t}\right)^{-\eta_e} Y^*_{f,t}, \tag{89}$$

where $\mathcal{E}_{h,t}$ represents real exports of home-produced goods to the foreign market, $P^*_{e,t}$ is the foreign-currency price of domestic exports, and P^*_t denotes the foreign aggregate price level. The term $\left(P^*_{e,t}/P^*_t\right)$ thus captures the relative price of domestic exports in units of foreign consumption. The parameter ω_e governs the steady-state export share, and η_e is the elasticity of foreign demand for home exports.

We assume producer-currency pricing for exports, so that the foreign-currency price of domestically produced exports is given by

$$P^*_{e,t} = \frac{P_{e,t}}{S_t}, \tag{90}$$

where $P_{e,t}$ is the domestic-currency price of exports and S_t denotes the nominal exchange rate.

We further impose the Law of One Price for domestically produced goods, implying

$$P_{e,t} = P_{d,t}. \tag{91}$$

Foreign demand for home exports, denoted by $Y^*_{f,t}$, follows an exogenous AR(1) process:

$$\log Y^*_{f,t} = (1 - \rho^*_y) \log \bar{Y}^*_f + \rho^*_y \log Y^*_{f,t-1} + \epsilon^*_{y,t}, \tag{92}$$

where $\bar{Y}^*_f$ denotes steady-state foreign demand for home exports, $\rho^*_y \in (0, 1)$ captures persistence, and $\epsilon^*_{y,t}$ is an exogenous foreign demand shock.

Finally, we impose the condition that the trade balance is fully reflected in changes in the foreign net asset position. The balance-of-payments identity is therefore given by

$$S_t B^*_{h,t} - S_t R^*_{t-1} \Gamma\left(B^*_{h,t-1}\right) B^*_{h,t-1} = P_{d,t} \mathcal{E}_{h,t} - P_{m,t} \mathcal{I}_{h,t}, \tag{93}$$

which states that net foreign asset accumulation equals the nominal trade balance.

3.9 Parameters and Calibration

We report the calibrated parameter values in Table 1. Since our small open economy E-DSGE model builds on the established international macroeconomic literature, most parameter choices follow standard calibrations.

The subjective discount factor, β, captures households' time preference. In our quarterly model, we set $\beta = 0.99$, which implies an annual real interest rate of approximately 4%. The parameter ψ_C denotes the coefficient of relative risk aversion (the inverse of the intertemporal elasticity of substitution). The literature typically calibrates this parameter between 1 and 3. We set ψ_L, the inverse of the

Frisch elasticity of labor supply, equal to 1, implying a moderate responsiveness of labor supply to wage changes. The labor disutility scale parameter, ϕ_L, is calibrated to 6.9164 to match a steady-state labor supply of one-third of total available time.

Turning to open-economy parameters, ω_C represents home bias (the degree of openness). Consistent with the small open economy literature, we set $\omega_C = 0.6$. The parameter ω_d, which captures the domestic expenditure share on brown goods, is set to 0.5 in line with recent open-economy studies. The elasticity of substitution between home and imported consumption goods, η_C, governs the responsiveness of domestic consumption to foreign goods, we set $\eta_C = 1.5$. Similarly, σ_d measures the elasticity of substitution between brown and green goods, and we calibrate it to 2, following standard practice. The parameter ν_L is set to 2, implying that labor allocation in the brown sector is twice as responsive as in the green sector.

Regarding production and nominal rigidities, we adopt conventional values widely used in the literature. The quarterly capital depreciation rate, δ, is set to 0.025. The capital share in production, α, is calibrated to 0.37. We introduce investment adjustment costs governed by γ_I, which is set to 15 following Annicchiarico and Di Dio (2015). Consistent with the same reference, the elasticity of substitution across varieties within each sector (brown or green) is set to 6. The Calvo price-stickiness parameter, ω, is calibrated to 0.5, and the inflation indexation parameter, ξ_h, is set to 0, implying no backward-looking indexation in price setting.

In the benchmark model, we assume an inflation-targeting monetary policy regime, which reflects the prevailing practice among many central banks worldwide. Following Smets and Wouters (2007), we set the interest rate smoothing parameter, ρ_r, to 0.74 and the inflation response coefficient, ϕ_π, to 1.5, ensuring that the Taylor principle is satisfied. In line with Annicchiarico and Di Dio (2015), we also include an output response coefficient, ϕ_y, to capture the sensitivity of the nominal interest rate to the output gap. We set $\phi_y = 0.1$, implying a relatively modest response to output fluctuations. This small value reflects a monetary policy framework primarily focused on inflation stabilization. In subsequent sections, we explore alternative parameter configurations and discuss how different calibrations correspond to the four main types of small open economy regimes considered in this study. For ease of exposition, however, we begin our analysis under the benchmark inflation-targeting regime and relax this assumption later.

As our framework is an Environmental-DSGE model, we incorporate a set of environment-related parameters into the small open economy structure. The parameter ρ_{emit} captures the persistence of emissions and is calibrated to 0.9, reflecting the high persistence typically observed in pollution dynamics. The parameters d_0, d_1, and d_2 govern the climatic damage function. Following Heutel (2012) and Annicchiarico and Di Dio (2015), we calibrate them to 1.3950×10^{-3}, -6.6722×10^{-6}, and 1.4647×10^{-8}, respectively. These values determine the nonlinear relationship between accumulated emissions and output losses. The abatement cost function is characterized by two parameters: ε_1 and ε_2, which control the scale and curvature of abatement costs, respectively. Consistent with the environmental macroeconomics literature, we set $\varepsilon_1 = 1.17$ and $\varepsilon_2 = 2.8$, implying increasing marginal costs of abatement. In the benchmark specification, we assume a predetermined carbon tax rate of 0.25, corresponding to a 25% tax on carbon emissions. In addition, following Annicchiarico and Di Dio (2015), we introduce the parameter ζ, which measures emissions intensity—i.e., the amount of carbon emissions generated per unit of output in the brown sector. We set $\zeta = 0.45$, a value commonly adopted in the literature.

As our framework is a small open economy DSGE model, several parameters governing foreign-sector dynamics must be calibrated. The risk-premium elasticity with respect to net foreign assets, ψ_b, is set to 0.01, a standard value that ensures stationarity of external debt. The price elasticity of export demand, η_e, is calibrated to 1.5, while the export demand shifter, ω_e, is set to 0.2. Foreign macroeconomic conditions follow AR(1) processes. We set the persistence parameters for foreign inflation, interest rates, and output to $\rho_\pi^* = 0.7$, $\rho_r^* = 0.9$, and $\rho_y^* = 0.8$, respectively, consistent with the high persistence typically observed in external shocks.

We incorporate three exogenous shocks into the model: a technology innovation shock, a domestic monetary policy shock, and a foreign monetary policy shock. The standard deviation of each shock is set to 0.01, consistent with the calibration in Schmitt-Grohé and Uribe (2003). These exogenous processes generate total factor productivity (TFP) shocks, domestic policy rate shocks, and world interest rate shocks, respectively.

[**Insert Table 1 Here**]

4 Impulse Responses and Steady State

We examine the dynamic effects of three external shocks: a positive total factor productivity (TFP) shock, a contractionary domestic monetary policy shock, and a contractionary global monetary policy shock. The impulse responses are generated under the benchmark specification of an inflation-targeting regime with a carbon tax rate of 0.0512.

We begin with the positive TFP shock. As shown in Figure 1, total consumption, investment, and output increase significantly following the shock. The mechanism is straightforward. Higher productivity lowers firms' marginal costs, leading retailers to reduce re-optimized prices. The resulting decline in the aggregate price level stimulates consumption. At the same time, the rise in the marginal product of capital increases the return to capital, encouraging investment. The joint expansion of consumption and investment drives output growth.

From an environmental perspective, total emissions rise while abatement effort declines. When productivity improves, firms face stronger incentives to expand production because the marginal products of labor and capital increase. Even in the presence of a carbon tax, firms reallocate resources from abatement toward production to exploit higher returns, leading to higher emissions. As emissions increase, carbon tax revenues also rise. In the small open economy setting, exports initially decline before rebounding and increasing, while imports rise persistently after the shock.

[**Insert Figure 1 Here**]

We next consider the contractionary domestic monetary policy shock, shown in Figure 2. A rise in the nominal interest rate leads to significant declines in total consumption, investment, labor demand, and output. Higher borrowing costs discourage firms from investing, which reduces capital accumulation and output, and lowers labor demand. The increase in the policy rate also raises the real interest rate, making savings more attractive and further depressing current consumption. At the same time, weaker economic activity reduces marginal costs, leading firms to lower prices and generating a decline in the aggregate price level.

From an environmental perspective, lower output results in lower total emissions. Abatement effort also declines, as firms scale back production and face weaker incentives to allocate resources to emission reduction. Although total emissions fall, reduced abatement implies that a larger share of emissions is taxed, which can increase carbon tax revenues. In the small open economy, exports decline markedly. Imports initially rise—reflecting an appreciation of the domestic currency following the monetary tightening—but subsequently fall as domestic income and demand weaken.

[**Insert Figure 2 Here**]

We conclude with a positive world interest rate shock (Figure 3). A rise in the global interest rate creates a return differential that drives capital outflows, tightening domestic financial conditions. Higher funding costs reduce investment in both brown and green sectors, leading to lower output and income, and consequently weaker aggregate consumption. Although consumption of brown and green goods briefly increases on impact, these effects dissipate quickly and turn negative.

In contrast, exports improve. The shock induces a depreciation of the domestic currency, making home goods relatively cheaper and boosting exports, while imports decline. Lower domestic production reduces total emissions, an effect reinforced by the presence of the carbon tax.

[**Insert Figure 3 Here**]

4.1 Steady-state Analysis: a Green Transition

To capture the transition effect of the small open economy, we examine steady-state outcomes under different carbon tax rates. This analysis warrants a clearer understanding of how the economic system responds to a change in the carbon tax rate, and thereafter the effectiveness of carbon policy. In a static setting, the steady-state results show a lasting change as a result of the introduction of environmental regulation. This exercise provides us with several policy scenarios which enables the comparison between the outcomes brought by different levels of carbon tax rate. We adopt seven rates ranging from 0 to 0.39, which covers non-policy case, low rate level, medium rate level, and high rate level. Our choice of these values aligns with real-world policy designs and existing literature in environmental economics.

The steady-state outcomes of the main macroeconomic variables are shown in Figures 4 and 5. First and foremost, we find that a higher carbon tax rate triggers a clear trend of green transition. By the green transition, we mean the aggregate demand and output in green sector have expanded, while the brown sector displays the opposite effects. Specifically, green sector's consumption, investment, labor recruitment, and output all experience rising trends as the carbon tax rate increases. By contrast, all of these variables show declining trends simultaneously. Moreover, a higher level of carbon tax rate reduces the total volume of carbon emission, while adding more tax transfers to household as a result of more tax revenues.

The mechanism behind the green transition that we observed in our static setting is self-revealing in Figures 4 and 5. As carbon taxes raise the total marginal cost of the brown production, the relative price of brown goods increases constantly. As pricier brown goods suppress the domestic brown demands, household substitutes brown consumption with more green consumption, as illustrated by the change of steady-state consumptions. The cost channel is the major contributor to the green transition since the carbon tax is supposed to elicit higher cost of consuming brown goods.

In the case of small open economy, we also note that both home imports and exports drop significantly as the tax rate increases. As a result, the imported consumption and total consumption both decrease. We reconcile this fact with the change of total demands in steady state. Specifically, the decline of brown demands exceeds the rise of green demands, pulling the aggregate demand down. As a result, home demands for foreign goods and home total final demands both inch down. This observation may uncover another side of the green transition: although more demands in green sector can be activated, the total aggregate demand and output may experience a negative effect. Therefore, policy makers may face the tradeoff between green transition and aggregate demand management.

[**Insert Figure 4 Here**]

[**Insert Figure 5 Here**]

4.2 Effectiveness of Carbon Taxes in a Dynamic Setting

One important merit of DSGE model is dynamic analysis for different external shocks to the economy. In this sub-section, we again target a benchmark model of inflation-targeting regime, while accounting for different values of carbon tax rate.

We purport to compare the dynamic macroeconomic outcomes between different taxes, which enables us to evaluate the effectiveness of the tax policy. We keep our carbon tax rates the same as the steady state case. The dynamic responses of the main macroeconomic variables are shown by Figure 6.

We consider a positive technology shock to the small open economy. In face value, a positive TFP shock can increase total consumption, investment, final output, and total demands. These changes indeed are documented by Figure 6. However, as we are comparing different carbon taxes and their relative effectiveness, some significant magnitude differences should be noted. In aggregate level, the higher the carbon tax, the lower the total consumption, consumption of home goods, consumption of foreign goods, and home imports. The magnitudes of these macrovariables experience clear and steady decline as the tax rate climbs up. However, the total emission is significantly decreased while the tax transfer is increasing as expected. Again, similar with what we recorded in the preceding section, there is a trade-off between emission reduction and economic outcomes. A higher tax is able to suppress the total emission, while the aggregate demands (including home imports of foreign goods) are also depressed.

Interestingly, we also find a positive change in the green sector. With a higher rate of carbon tax, the magnitudes of green consumption, investment, labor hiring, and output all climb up significantly. The brown sector is accordingly receiving a clear decrease in these macrovariables. In the short term, this pattern is showing a clean and significant diverging effect between brown sector and green sector. Although all variables in both sectors are increasing as a result of positive TFP shock, the magnitudes of the increase are directionally opposite between brown and green sectors.

[**Insert Figure 6 Here**]

4.3 Effective of Carbon Taxes across Monetary Regimes

We have documented the diverging pattern between brown and green sectors for the inflation-targeting regime. However, as we are investigating the different macroeconomic responses of the four types of monetary policy regime of small open economy, it is imperative to analyze how different monetary policy regimes react to an introduction of the carbon tax. Currently, we have shown the significant divergent outcomes when the carbon tax changes. Nevertheless, to fill the general picture of our research, we in this section draw a comparison across different monetary policy regimes given a carbon tax rate of 5.12%, whose value is aligned with the existing literature of Annicchiarico and Di Dio (2015).

Figures 7 and 8 display impulse responses of different monetary-exchange rate policy regimes that we consider given a positive TFP shock. We find that the fixed peg exchange rate regime shows clearly higher level of variation in both direction and magnitude, compared with the other three types monetary regimes. For example, although the fixed rate regime presents the highest increment in green consumption, its brown consumption also inches up significantly. There are also notable fluctuations in demand of home goods and home exports. Importantly, the increase in total emission of fixed exchange rate regime is the smallest among all types of monetary policy regimes.

By contrast, the managed float exchange rate regime seems to deliver the most stable changes in consumptions, investments, and output. For instance, the increases in green consumption and brown consumptions are very stable, without notable fluctuations which can be monitored in fixed peg regime and inflation-targeting regime. This observation can also extend to labor hiring and outputs in brown and green sector. Although the change of magnitudes in dual-targeting policy regime is also stable, the managed float policy regime displays more stable reactions in key variables such as brown and green consumptions which are our main focus.

We also inject a negative global interest rate shock into the economy as the small open economy is prone to be impacted by the changes of global credit condition. The trend we documented is similar with the case of positive TFP shock but with some deviations. First and foremost, we again note that the managed float exchange rate regime delivers the most stable variation of change in most macroeconomic variables. We find very stable reactions of increase in total home consumptions, brown consumption, green consumption, brown investment, and green investment. Particularly, the home emission drop in managed float exchange rate regime is stable but constant, when compared with three other policy regimes. Second, the contractionary monetary policy shock ignites largest contractions in total demand and final output for fixed exchange rate policy regime. Both the brown and green outputs are facing highest level of deterioration for this regime. Moreover, the drops investments and consumptions in brown and green sectors are the largest relative to the other three regimes.

[**Insert Figure 7 Here**]

[**Insert Figure 8 Here**]

4.4 Decompositions of Effectiveness of Carbon Tax Policy across Policy Regimes

We have documented the clearly different reactions of macroeconomic variables across different monetary policy regimes. In order to check our evidence and fortify the robustness of our findings, we combine carbon tax change and monetary policy regime together in this section. Specifically, we show a carbon tax change to each monetary policy regime under a positive TFP shock. This exercise can be seen as a sensitivity analysis of evaluating the effectiveness of carbon tax policy and how it differs between different monetary policy regimes. The results are reported in Figures 9 to 12.

First, we compare two carbon tax rates: 5.12% and 25%, which shows a lower level and a medium level of the carbon tax rate. Again, under a positive TFP shock, all macroeconomic variables move as expected. For example, across four policy regimes, the total emission increases, aggregate demands and outputs rise, the final price of green good decreases, and the final price of brown good increases. Moreover, when carbon tax is higher (25%), we continually notice the diverging in brown and green sectors, as we discussed in the preceding sections. For instance, the increases in green consumption, investment, and output are larger, while the brown sector experiences the opposite effects.

More importantly, across different types of monetary policy regime, we find evidence that managed float exchange rate regime delivers the most stable changes after a tax change. When the carbon tax rate increases, the changes of macrovariables for managed float exchange regime show less fluctuations, while the fixed exchange rate policy regime displays the highest variation after the tax increases from 5.12% to 25%. This phenomenon reflects what we documented before, the managed float regime produces stable macro-level changes, while the fixed rate regime is more volatile when there is a tax change.

[**Insert Figure 9 Here**]

[**Insert Figure 10 Here**]

[**Insert Figure 11 Here**]

[**Insert Figure 12 Here**]

4.5 Effectiveness of Cap-and-Trade Policy

As discussed before, we prompt to discuss the effectiveness of environmental policies that are widely adopted by small open economies to counter the carbon emission. In the preceding sections, we emphasize and evaluate how macroeconomic variables react to the carbon tax policy in both the static setting and dynamic setting. However, another broadly used environmental instrument cap-and-trade also deserves a careful analysis, since it is within the usually applied environmental policy toolkit in real-world policy making. In this subsection, we fully introduce the two main environmental policy instruments: carbon tax and cap-and-trade. For each policy vehicle, we define its meaning and draw the connection between the policy tool and our research purpose.

We acknowledge that there are some other policy tools such as intensity target and eco-labeling that we do not consider. However, the two regulation conducts we select here generally represent the most broadly applied instruments in addressing environmental pollution.

The carbon tax policy serves as a regulatory vehicle to discourage polluting firms from releasing voluminous carbon dioxide (CO2) by charging a fee on each ton CO2 emission. For example, Canada set a price of CA$ 80 per ton of CO2 and equivalent, which is also expected to rise to CA$170 per ton in 2030. Therefore, firms that generating carbon dioxide need to pay a price for the amount of CO2 they produce.

Cap-and-trade tool is utilized by government agencies to determine a capped level of total emission from covering firms. Then, firms obtain permits/allowances for releasing CO2 during their respective production processes. As some firms may lower the emission while others may need more allowances for emission, there exists a carbon trading market for firms to trade their pollution permits. This policy design is well exemplified by the European Union Emission Trading System (EU-ETS) and California's Cap-and-Trade Program. Thus, firms can trade their pollution permits according to their operational demands under a fixed/capped level of total emission.

We now formally express the two environmental policies that will be fitted into our DSGE model framework.

As we have already defined carbon price $P_{z,t}$, total emission $Emit_t$, emission flow $(1-\kappa_{h,t})\zeta X_{h,t}$, abatement effort $\kappa_{h,t}$, and output level $X_{h,t}$, the relevant policy regimes can be stated as:

·Carbon Tax: the government levies a (nominal) carbon price $P_{z,t}$ on emission from current period, $(1 - \kappa_{h,t})\zeta X_{h,t}$, before which the firms have abated some emission within the period by putting into abatement effort $\kappa_{h,t}$. The carbon tax rate (real carbon price), τ, equals $\frac{P_{z,t}}{P_{h,t}}$, which is set at a constant value.

·Cap-and-Trade: the government sets a predetermined total emission cap within current period $\overline{(1 - \kappa_{h,t})\zeta X_{h,t}}$. Define $\bar{M}_{h,t} = \overline{(1 - \kappa_{h,t})\zeta X_{h,t}}$ as the capped emission. Firms with emission allowances participate in a carbon trade market which decides the (nominal) carbon price $P_{z,t}$. Notice that the total emission $Emit_t$ is changed to: $Emit_t = \rho_{emit} Emit_{t-1} + \bar{M}_{h,t}$. Use the new emission stock to re-calculate firms' profit maximization problems.

We use cap-and-trade policy as a policy instrument comparable to carbon tax policy. Again, similar to what we did in carbon tax analysis, we undertake steady-state analysis and macroeconomic responses in a static setting and a dynamic setting, respectively. Figures 13-16 show the changes of main macrovariables with the introduction of cap-and-trade policy and different types of small open economy. We compile our findings as follows.

First, as to the static setting, the green transition is illuminating which confirms our conjecture that cap-and-trade policy tool can bring similar green trend as what we observed in carbon tax case. Similar with the results of the carbon tax, the green sector's consumption, investment, labor hiring, and output all experience a significant hike after the cap-and-trade level increases. By contrast, the brown sector's consumption, investment, labor hiring, and output all see a clear decline. At the aggregate level, same as the carbon policy case, we note that the total demands of home consumption, home exports, and home imports drop significantly when the capped level of emission is higher. This suggests that a downside effect on aggregate demand is existent along with the green transition. There is a trade-off between the application of a higher capped level of emission and the containment of domestic aggregate demands. We also notice that the variations of these variables facing the cap-and-trade policy are smaller than that of carbon tax policy case. This result aligns with the existing environmental economics literature that documents a lower variation of change produced by the cap-and-trade policy.

Second, concerning the dynamic setting, we impose a positive TFP shock to the economy. Given this external shock, we compare and contrast dynamic reactions to a series of capped level of emission ranging from 100% to 70%. Figure 13 to Figure 15 show the impulse responses of a set of main macroeconomic variables facing different levels of capped emission. In face value, the TFP shock induces positive changes in total consumption, consumption of home goods, consumption of imported goods, investments, exports, imports, and outputs. This result is well-established in the existing DSGE literature. When diving into the differences between the brown sector and green sector, we again find the diverging effect which is also recorded in the case of carbon tax policy. As the capped level of emission climbs up, the consumption, investment, labor hiring, and output in the green sector all experience larger increases in magnitude, while these variables in the brown sector sees the opposite effects. These findings well confirm the effect of cap-and-trade policy in depressing brown-side demand and eliciting green-side demand.

Last but not least, we introduce a pre-determined capped level of emission and compare the impulse responses of macroeconomic variables across different types of monetary policy regime. We inject a positive TFP shock to the economy, and contrast 90% and 80% levels of impulse responses given this external shock. Again, we observe the positive responses of demand of home goods, foreign goods, domestic investment, and final total output in all four types of monetary policy regime. However, these variations of responses are lower than that of carbon tax policy case. We also find that the fixed exchange rate policy regime delivers the highest level of variation in consumption of home goods, consumption of foreign goods, brown consumption, green consumption, and exports. By contrast, the managed float policy regime seems to produce relatively lowest fluctuation in home imports, consumptions, and labor hiring, though this effect is not strongly significant. In general, we still find notable evidence that home exports and imports reflect most stable change among all the four monetary policy regimes. These observations are well aligned with our previous finding that the managed float regime shows relatively stable outcomes while the fixed exchange rate regime delivers the largest fluctuation of the macrovariables that we focus on.

[**Insert Figure 13 Here**]

[**Insert Figure 14 Here**]

[Insert Figure 15 Here]

5 Welfare Analysis

5.1 Deterministic Welfare across Monetary Policy Regimes and Carbon Pricing Stringency

In this subsection and the subsequent sections follow, we conduct a set of welfare analysis for different types of monetary policy regimes. As a well-established tradition in economics literature, we mainly implement two types of welfare evaluation: deterministic welfare and dynamic welfare. These exercises enable us to compare and contrast the welfare effects of different policy regimes. Specifically, we use consumption equivalence change to inform our welfare comparison, which helps us to derive the quantitative meanings of welfare effects across different monetary policy regimes.

We begin our analysis with the deterministic welfare outcome. We compile results of the four policy monetary policy regime. In this case, as we target a steady-state nominal interest rate, the steady-state rate is the same across different monetary policy regimes. As a result, the inter-temporary Euler equation produces the same steady-state consumption and accordingly the same consumption change in welfare analysis. Tables two to three compile the deterministic welfare outcomes in the four monetary policy regimes. Clearly, as carbon tax rate increases, the steady-state welfare change displays the same effects for all monetary policy regimes. The higher the carbon tax rate, the larger welfare loss, suggesting that a more assertive carbon tax policy can create higher welfare losses, which applies equally to all the regimes. More specifically, above a higher carbon tax level (larger than 25% rate), the magnitude of welfare losses expands more quickly. This means if the policy makers choose a higher starting rate of carbon tax, then the increment in the tax can more aggressively induce larger welfare losses. Tables 2 to 5 display the change of static welfare in relative to carbon tax rate, which affirms our finding that a higher tax rate can suppress welfare which may contract more quickly when the rate climbs higher.

[Insert Table 2 Here]

[Insert Table 3 Here]

[**Insert Table 4 Here**]

[**Insert Table 5 Here**]

5.2 Carbon Tax Policy: Dynamic Welfare across Different Monetary Policy Regimes

We now turn to a dynamic analysis of welfare change among the four monetary policy regimes that we survey. Again, we adopt the consumption equivalence variation (CEV) method to reflect the welfare changes. Although the higher rate of carbon tax surely suppresses the welfare gains, there are significant deviations in the case of dynamic setting. Different monetary policy regimes show distinctions of welfare effect after the increase in carbon tax. If we rank the regimes by welfare losses, the managed float policy regime delivers the lowest welfare, seconded by the dual-targeting policy regime, then the inflation-targeting regime, with the fixed exchange rate policy regime produces the highest loss of welfare. Particularly, the managed float exchange rate regime shows the lowest welfare loss both in the lower range of the taxes and the higher range of the taxes. By contrast, the fixed exchange rate policy regime not only experiences a sharp loss of welfare when the carbon tax is very small, but also encounters highest loss of welfare when the rate is raised significantly.

Our findings above can be reconciled by the existence of the export-side cushion effect. The managed float small open economy, for example, can have relatively more space to adjust the home currency value relative to the foreign currency. A higher value of foreign currency would push the monetary authorities of this regime to according adjust the domestic nominal interest rate rather than having to strictly follow the rate of foreign rate. As the authority of managed float regime is tolerant to a depreciation of home currency within a policy band, the lower value of home currency can stimulate more domestic exports to the world market. This channel of export effect may be strong enough to maintain the aggregate output and total revenues of domestic household. By contrast, the fixed exchange rate regime has to closely follow the change of world policy rate, which can push up the domestic interest rate. As a result, a deterioration of current account cancels out the increase of home output and household revenues, which can produce higher welfare losses.

Tables 6 to 9 document the dynamic welfare change relative to the carbon taxes in different monetary policy regimes. Again, the change of magnitudes differ significantly between policy authorities. As we discussed above, the managed float exchange rate regime exhibits the lowest welfare loss in both the lower range of carbon tax and higher range of carbon tax. On the contrary, the fixed exchange rate policy regime clearly delivers the highest level of welfare losses in almost each segment of carbon tax change.

[**Insert Table 6 Here**]

[**Insert Table 7 Here**]

[**Insert Table 8 Here**]

[**Insert Table 9 Here**]

5.3 Cap-and-trade Policy: Dynamic Welfare Analysis across Different Monetary Policy Regimes

In this section, we bring back the dynamic analysis for cap-and-trade policy instrument. The tables of welfare results (Tables 10 to 13) and trends of welfare changes are reported below accordingly. We again set a series of capped level of emission starting from 100% (no cap) to 70% (high cap) in order for understanding how capped level affects the welfare change in dynamic setting across the four types of monetary policy regimes.

To rank the welfare outcomes, we again use the consumption equivalence variation to represent the welfare loss. The finding is illuminating an revealing: the managed float exchange rate regime displays the smallest change of welfare loss as the capped level of emission rises. The magnitude of welfare loss is followed by the inflation-targeting regime and dual-targeting regime, with the fixed exchange rate regime again delivers the largest loss of dynamic welfare. This set of results is well in line with what we found in the case of carbon tax policy. The managed float policy regime again is the most welfare amenable while the fixed exchange rate policy produces the worst performances according to the welfare effect.

We attribute our findings to the effectiveness of monetary policy in stimulating the aggregate demand. As managed float exchange rate regime is able to float the rate relatively rather than rigidly follow the increase of policy rate in the case of fixed exchange rate regime, the policy makers can induce the depreciation of home currency which can in turn stimulate more foreign demands of home output. Even if the higher capped level of emission is indeed suppressing the home brown product and brown aggregate demands, the export channel is working well enough to stimulate more exports which can support the improvement of home currency account and the home total revenues as a result.

[**Insert Table 10 Here**]

[**Insert Table 11 Here**]

[**Insert Table 12 Here**]

[**Insert Table 13 Here**]

5.4 Comparison between Carbon Tax and Cap-and-Trade

In order to compare the policy effectiveness of carbon tax and cap-and-trade, we match each capped emission flow value to a respective carbon tax rate. For example, if we have a 85% capped level of emission, by calculation of steady state outcome, the equivalent value of carbon tax rate is accordingly 0.168. We do this calculation for each capped level and carbon tax rate. For capped emissions in our research (95%,90%,85%,80%,75%,70%), the equivalent carbon tax rates are respectively: 0.085, 0.124, 0.168, 0.217, 0.271, 0.332.

After constructing the equivalence of capped level of emission and carbon tax rate, we re-obtain the dynamic welfare for each monetary-exchange rate policy regimes, whose results are attached below.

[**Insert Table 14 Here**]

[**Insert Table 15 Here**]

[**Insert Table 16 Here**]

[**Insert Table 17 Here**]

In all comparable level of welfare effects in different monetary policy regimes, we find that the cap-and-trade policy delivers the lowest welfare loss when compared with the same level of carbon tax policy. Although the managed float exchange rate regime continually produces smallest welfare loss in both cap-and-trade and carbon tax policies, the welfare gain in cap-and-trade is significantly higher than that of carbon tax policy. Even for the fixed exchange rate regime which brings the highest welfare loss, the loss of cap-and-trade policy is relatively lower than that of carbon tax policy. These exercises of comparing the carbon tax policy and cap-and-trade policy are in line with the findings of current literature that the cap-and-trade policy is relatively performing better in dynamic welfare evaluation.

6 Conclusion

This paper studies the interaction between environmental policy and monetary - exchange rate regimes in a small open economy. We develop a New Keynesian E-DSGE model with heterogeneous brown and green sectors and embed two canonical carbon-pricing instruments, carbon taxes and cap-and-trade, into an open-economy framework with alternative monetary arrangements. The model integrates standard open-economy features with endogenous emissions and abatement decisions, allowing for a unified analysis of transition dynamics, macroeconomic volatility, and welfare.

Three main conclusions emerge. First, both carbon taxes and cap-and-trade policies induce a structural green transition. As carbon pricing tightens, green-sector consumption, investment, employment, and output expand, while brown-sector activity contracts. This reallocation mechanism is robust across all monetary regimes. Moreover, under external shocks such as productivity or monetary disturbances, stronger carbon pricing amplifies green-sector responses and accelerates brown-sector contraction.

Second, monetary–exchange rate regime choice plays a critical role in shaping macroeconomic stability during the transition. For a given environmental policy, a fixed exchange rate peg generates the largest fluctuations in output and trade, whereas a managed float delivers the most stable responses. Inflation targeting and dual targeting lie between these extremes. Although stricter environmental policies reduce emissions under all regimes, the associated adjustment costs differ substantially across monetary frameworks.

Third, welfare comparisons indicate that cap-and-trade consistently entails smaller welfare losses than an equivalent carbon tax. Across monetary regimes, the managed float minimizes welfare losses as environmental policy tightens, while the fixed peg performs worst. Taken together, the results suggest that for small open economies pursuing de-carbonization, exchange rate flexibility enhances macroeconomic stability and improves welfare outcomes when combined with carbon pricing.

References

Acemoglu, D., Aghion, P., Bursztyn, L., and Hemous, D. (2012). The environment and directed technical change. *American economic review*, 102.1:131–166.

Adolfson, M., Laséen, S., Lindé, J., and Villani, M. (2008). Evaluating an estimated new keynesian small open economy model. *Journal of Economic Dynamics and Control*, 32:2690–2721.

Akhtaruzzaman, M. and Rahman, M. (2024). Commonality in systemic risk from green and conventional energy. *Energy Economics*, 12.

Alba, J., Liu, J., Chia, W. M., and Park, D. (2020). Foreign output shock in small open economies: A welfare evaluation of monetary policy regimes. *Economic Modelling*, 86:101–116.

Al-Ghussain, L. (2019). Global warming: review on driving forces and mitigation. *Environmental Progress and Sustainable Energy*, 38.1:13–21.

Angelopoulos, K., Economides, G., and Philippopoulos, A. (2010). What is the best environmental policy? taxes, permits and rules under economic and environmental uncertainty. *CES IFO WORKING PAPER*, NO . 2980:1–30.

Annicchiarico, B. and Di Dio, F. (2015). Environmental policy and macroeconomic dynamics in a new keynesian model. *Journal of Environmental Economics and Management*, 69:1–21.

Annicchiarico, B. and Diluiso, F. (2019). International transmission of the business cycle and environmental policy. *Resource and Energy Economics*, 58:101112.

Bajzik, J., Havranek, T., Irsova, Z., and Schwarz, J. (2020). Estimating the armington elasticity: The importance of study design and publication bias. *Journal of International Economics*, 127.C.

Bauer, M. and Swanson, E. (2023). A reassessment of monetary policy surprises and high-frequency identification. *NBER Macroeconomics Annual*, 37.1:87–155.

Beltran, D. and Draper, D. (2008). Estimating the parameters of a small open economy dsge model: Indentifiability and inferential validity. *Board of Governors of the Federal Reserve System: International Finance Discussion Papers*, 955:1–39.

Besley, T. and Persson, T. (2025). The political economics of green transitions. *The Quarterly Journal of Economics*, 138:1863–1906.

Bocola, L. and Lorenzoni, G. (2020). Financial crises, dollarization, and lending of last resort in open economies. *American Economic Review*, 110.8:2624–2557.

BoI (2025). Bank of israel monetary policy. *Bank of Israel*.

BoK (2025). Bank of korea monetary policy article 1. *Bank of Korea*.

Bridgman, B., Duernecker, G., and Herrendorf, B. (2018). Structural transformation, marketization, and household production around the world. *Journal of Development Economics*, 133:102–126.

Buntaine, M. T., Greenstone, M., He, G., Liu, M., and Wang, S. (2024). Does the squeaky wheel get more grease? the direct and indirect effects of citizen participation on environmental governance in china. *American Economic Review*, 114(3):815–850.

Bürgisser, R., Stadelmann-Steffen, I., and Armingeon, K. (2024). Can information, compensation and party cues increase mass support for green taxes? *Journal of European Public Policy*, 25:1–28.

Cho, H. and Ackom, E. (2025). Artificial intelligence (ai)-driven approach to climate action and sustainable development. *Nature Communications*, 16:1228.

Christiano, L. and Trabandt, M.and Walentin, K. (2011). Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control*, 35(12):1999–2041.

Cooley, T. F., editor (1995). *Frontiers of Business Cycle Research.* Princeton University Press, Princeton, NJ.

Dong, F., Mei, D., and Xiao, Z. (2025). Asset bubbles and financial frictions in small open economies. *Journal of International Money and Finance*, 150.

Etukudoh, Umoh, H., Ibekwe, and Sonko (2024). Carbon emission reduction strategies: A global policy review and analysis of effectiveness. *Economic Growth and Environment Sustainability*, No.2:69–75.

Fang, S., Rahaman, M., Bharti, J., Reisner, E., Robert, M., Ozin, G., and Hu, Y. (2023). Photocatalytic co2 reduction. *Nature Reviews Methods Primers*, No.1:61.

Ferrari, A. and Landi, V. (2024). Will the green transition be inflationary? *IMF Economic Review*, 20:1–64.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications.* Princeton University Press.

Gali, J. and Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, 72(3):707–734.

Gatti, L. V., Basso, L., Miller, J., Gloor, M., Gatti Domingues, L., Cassol, H., and Tejada, G. e. a. (2021). Amazonia as a carbon source linked to deforestation and climate change. *Nature*, 595:7867.

Ghosh, A., Ostry, J., and Quershi, M. (2015). Exchange rate management and crisis susceptibility: A reassessment. *IMF Economic Review*, 63.1:238–276.

Goldberg, P. and Reed, T. (2023). Is the global economy deglobalizing? if so, why? and what is next? *Brookings Papers on Economic Activity*, No.1:347–423.

Heutel, G. (2012). How should environmental policy respond to business cycles? optimal policy under persistent productivity shocks. *Review of Economic Dynamics*, 15:244–264.

HKMA (2025). Hong kong monetary authority monetary policy. *Hong Kong Monetary Authority*.

Hong Kong Government (2026). Climate change. https://www.gov.hk/en/residents/environment/global/climate.htm.

Ilzetzki, E., Reinhart, C., and Rogoff, K. (2021). Rethinking exchange rate regimes. *NBER Working Paper*, No.29347:1–66.

International Carbon Action Partnership (2026). Korean emission trading system. https://icapcarbonaction.com/en/ets/korea-emissions-trading-system-k-ets.

Irwin, D. (2025). Trade policy, exchange rates, and the globalization surge of the 1990s. *The Journal of Economic History*, 85.2:303–335.

Justiniano, A. and Preston, B. (2010). Can structural small open-economy models account for the influence of foreign disturbances? *Journal of International Economics*, 81:61–74.

Kazinnik, S. and Papell, D. (2021). Monetary policy rules in practice: The case of israel. *International Review of Economics and Finance*, 76:308–320.

Kolasa, M., Ravgotra, S., and Zabczyk, P. (2025). Monetary policy and exchange rate dynamics in a behavioral open economy model. *Journal of International Economics*, 155:104087.

Kolcava, D. (2023). Greenwashing and public demand for government regulation. *Journal of Public Polocy*, 43(1):173–198.

Kosnik, L.-R. (2018). Cap-and-trade versus carbon taxes: which market mechanism gets the most attention? *Climatic Change*, 151, No.3:605–618.

Lenton, T., Jess, C. X., Abrams, A. G., Loriani, S., Sakschewski, B., and Zimn, C. e. a. (2023). Quantifying the human cost of global warming. *Nature Sustainability*, No.10:1237–1247.

Lozej, M., Onorante, L., and Rannenberg, A. (2023). Countercyclical capital regulation in a small open economy dsge model. *Macroeconomic Dynamics*, 27(5):1230–1267.

MacDonald, R. and Taylor, M. (1993). The monetary approach to the exchange rate: rational expectations, long-run equilibrium, and forecasting. *Staff Papers*, 40.1:89–107.

MAS (2025). Monetary authority of singapore monetary policy. *Monetary Authority of Singapore*.

Massazza, A., Alonzo, N., Augustinavicius, J., Batz, C. S., Beagley, J., Carvalho, L. R., Daniel, M., Fleury, J., Guluzade, N., Hijazi, Z., Lawrance, E. L., el Omrani, O., Uakkas, S., Ugo, V., and Umesh, H. (2025). Why mental health should be embedded across climate and health discussions at cop29. *Lancet Psychiatry*, 12(1):9–11.

Minesso, M. and Pagliari, M. (2023). No country is an island. international cooperation and climate change. *Journal of International Economics*, 145:1–17.

Mzoughi, H., Urom, C., Salah, G., and Guesmi, K. (2022). Downside and upside risk spillovers between green finance and energy markets. *Finance Research Letters*, 47.

National Climate Change Secretariat (2026). Carbon tax. https://www.nccs.gov.sg/singapores-climate-action/mitigation-efforts/carbontax/.

Ngai, R. and Pissarides, C. (2008). Trends in hours and economic growth. *LSE Research Online Documents on Economics*, 3828.

Obstfeld, M. and Zhou, H. (2022). The global dollar cycle. *Brookings Papers on Economic Activity*, No.2:361–447.

Ploeg, V., Frederick, A. R., and Reanos, M. (2022). Gathering support for green tax reform: Evidence from german household surveys. *European Economic Review*, 141.

Schmitt-Grohé, S. and Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, 61:163–185.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97:586–606.

Smiech, S., Papiez, M., and Dabrowski, M. (2024). Output volatility and exchange rates: New evidence from the updated de facto exchange rate regime classifications. *International Review of Economics and Finance*, 89:894–908.

Tyquin, E., Weeks, C., Mehta, A., Newton, C., and Burke, N. (2024). Delivering an energy export transition: Impact of conflicting and competing informational contexts on public acceptance of australia's hydrogen export industry. *International Journal of Hydrogen Energy*, 61:226–237.

UN (2025). Leaders' session on climate and the just transition. *United Nation Climate Action.*

UN (2026). The paris agreement. *United Nation Climate Change.*

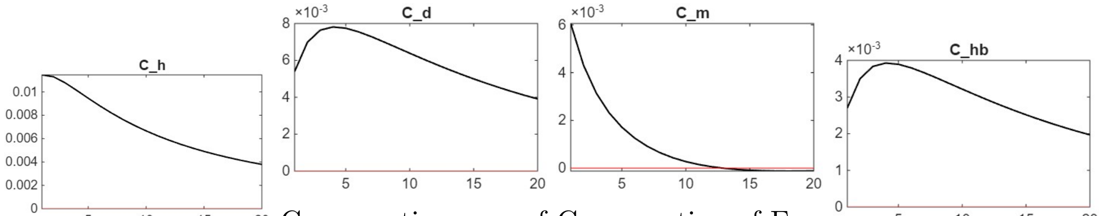
Wu, T., Wen, L., and Yi, M. (2024). Balancing growth targets and environmental regulations: An empirical analysis of dual policy impact on corporate environmental responsibility–insights from china. *Journal of Environmental Management*, 355:120500.

Zhou, J.-F., Wu, D., and Chen, C. (2022). Cap and trade versus carbon tax: an analysis based on a cge model. *Computational Economics*, 59, No.2:853–885.

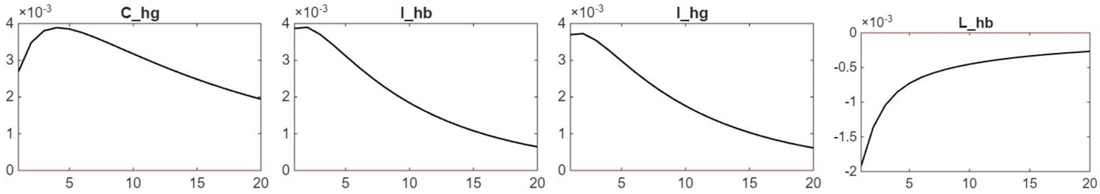
Tables and Figures

Figures

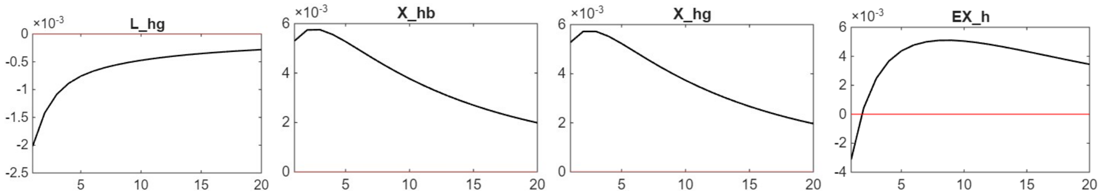
a



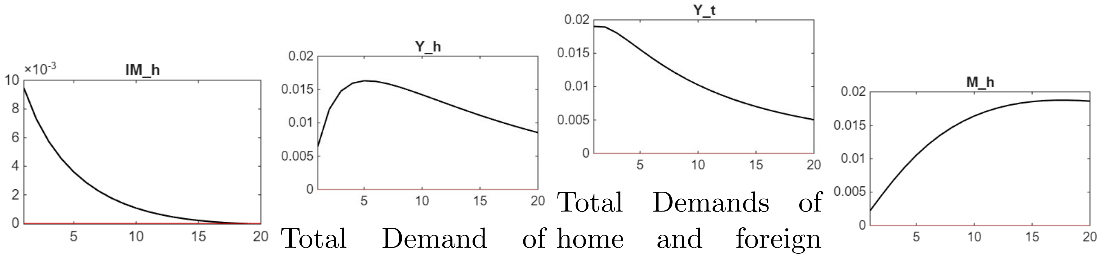
Total Consumption
Consumption of Home Goods
Consumption of Foreign Goods
Brown Consumption



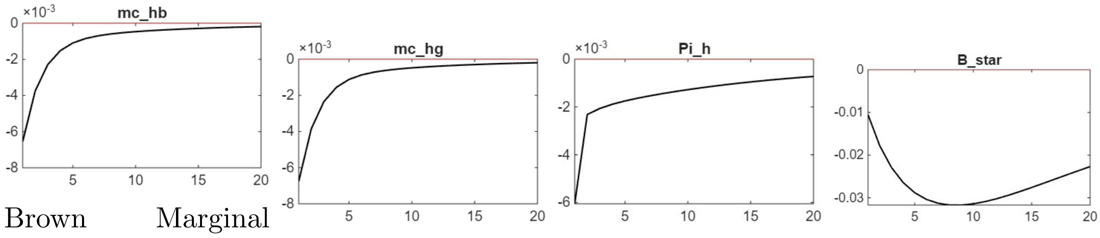
Green Consumption
Brown Investment
Green Investment
Brown Labor



Green Labor
Brown Output
Green Output
Home Export



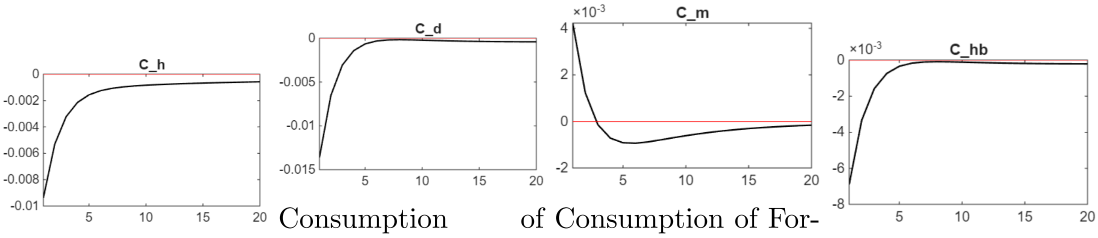
Home Import
Total Demand of Home goods
Total Demands of home and foreign Goods
Home Emission



Brown Marginal Cost
Green marginal Cost
Home Inflation
Foreign Net asset

Figure 1: Impulse responses to a positive technology shock

b

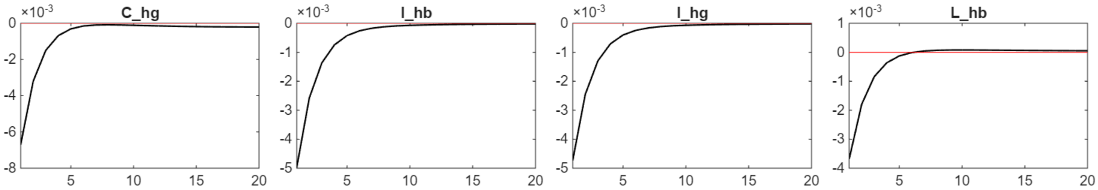


Total Consumption

Consumption of Home Goods

Consumption of Foreign Goods

Brown Consumption

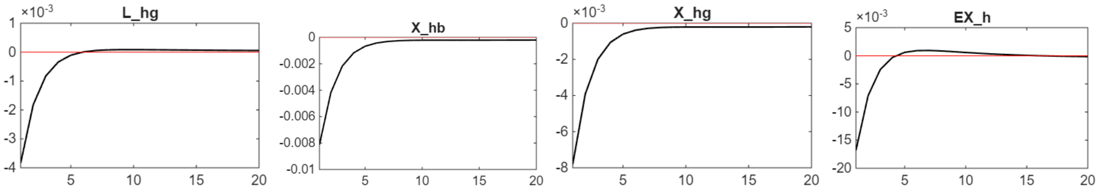


Green Consumption

Brown Investment

Green Investment

Brown Labor

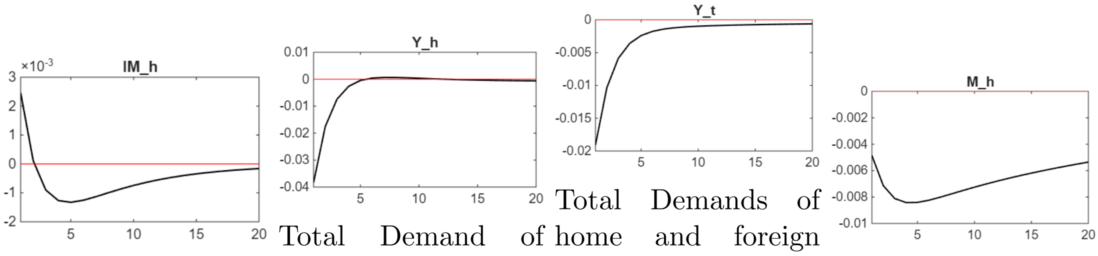


Green Labor

Brown Output

Green Output

Home Export

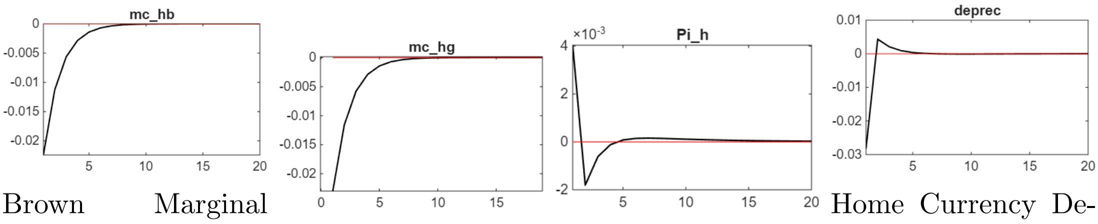


Home Import

Total Demand of Home goods

Total Demands of home and foreign Goods

Home Emission



Brown Marginal Cost

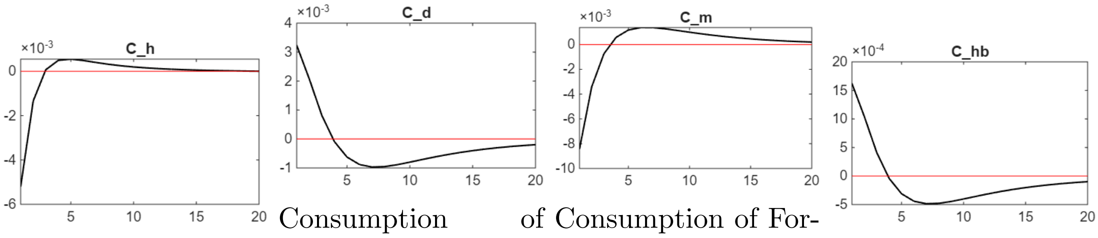
Green marginal Cost

Home Inflation

Home Currency Depreciation

Figure 2: Impulse responses to a contractionary monetary shock

c

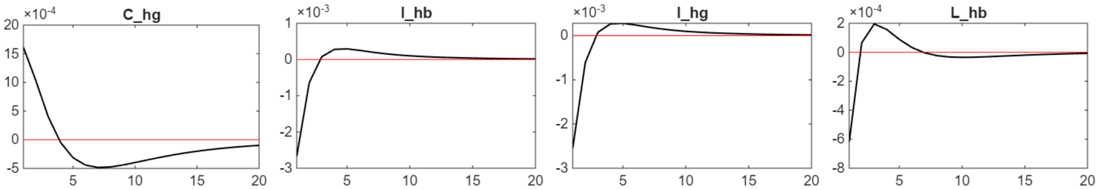


Total Consumption

Consumption of Home Goods

Consumption of Foreign Goods

Brown Consumption

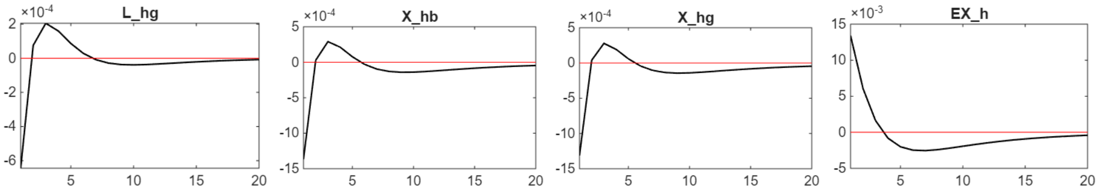


Green Consumption

Brown Investment

Green Investment

Brown Labor

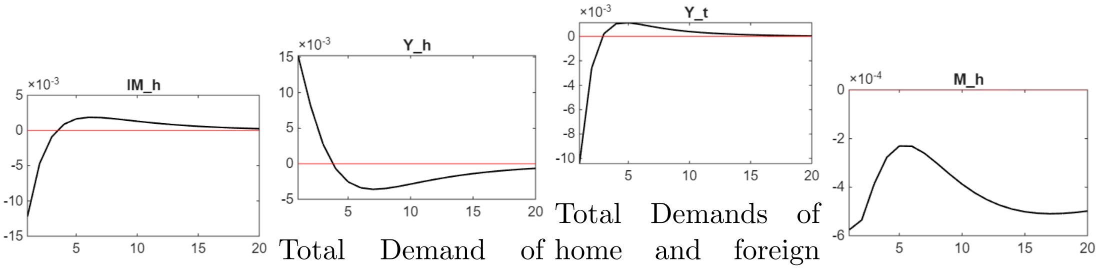


Green Labor

Brown Output

Green Output

Home Export

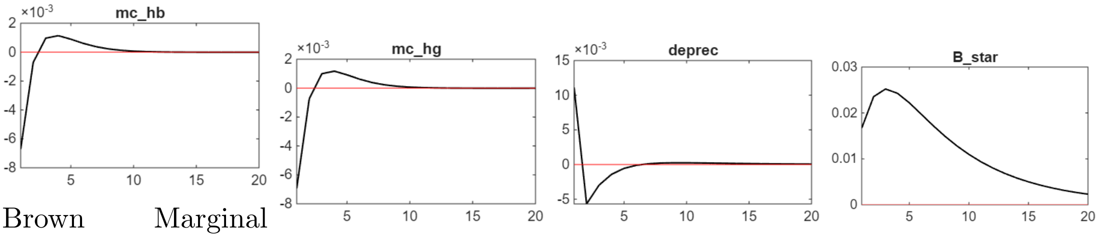


Home Import

Total Demand of Home goods

Total Demands of home and foreign Goods

Home Emission



Brown Marginal Cost

Green marginal Cost

Home Inflation

Foreign Net asset

Figure 3: Impulse responses to a contractionary global interest rate shock

Figure 4: Steady State Comparisons of Different Carbon Tax Rates

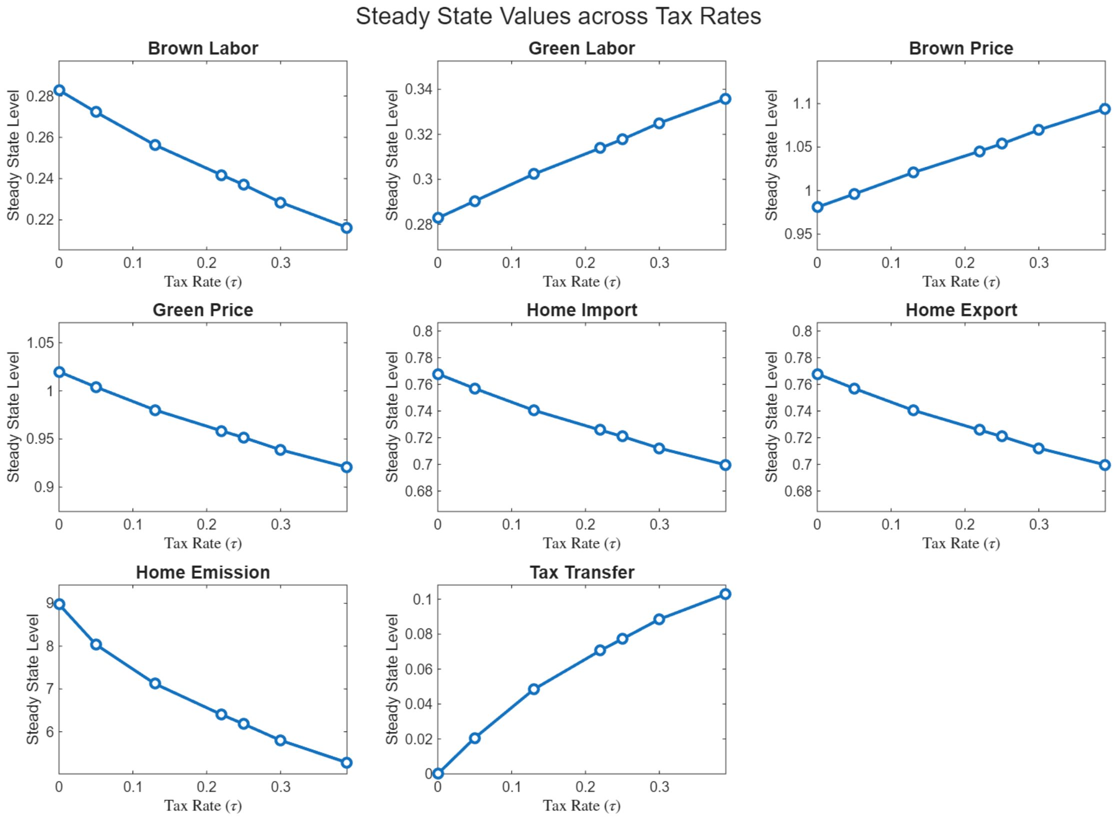


Figure 5: Steady State Comparisons of Different Carbon Tax Rates

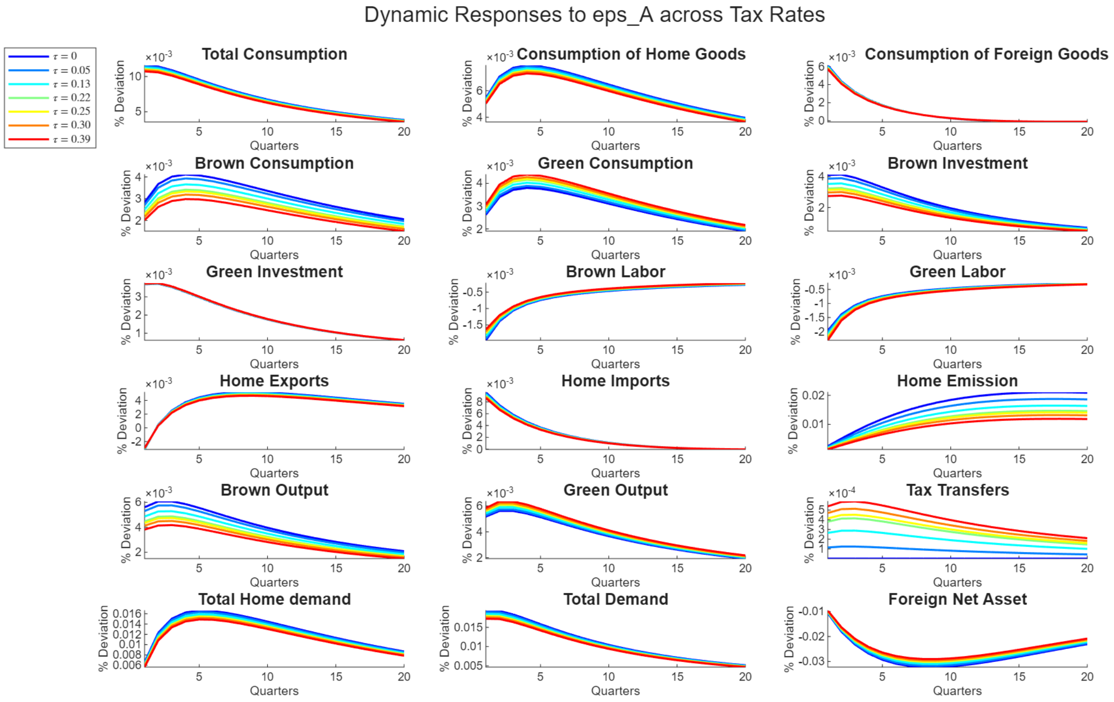


Figure 6: Impulse Responses of Different Carbon Tax Rates: Positive TFP Shock

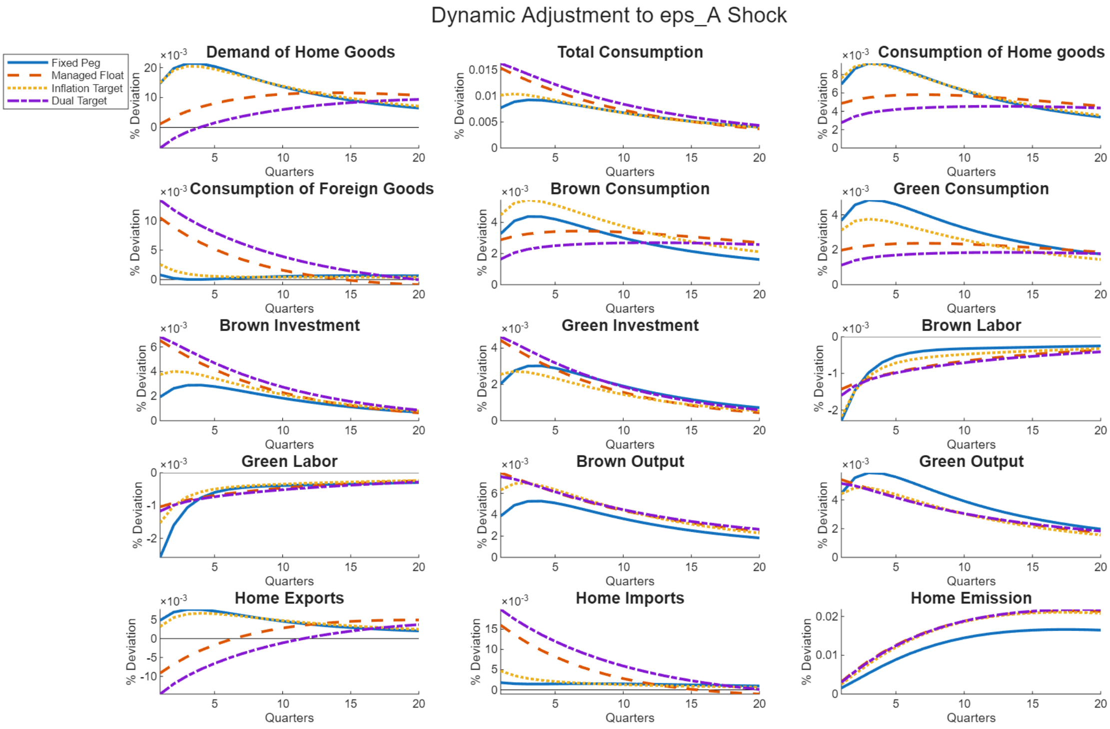


Figure 7: Impulse Responses of Different Monetary and Exchange Rate Regimes: Positive TFP Shock

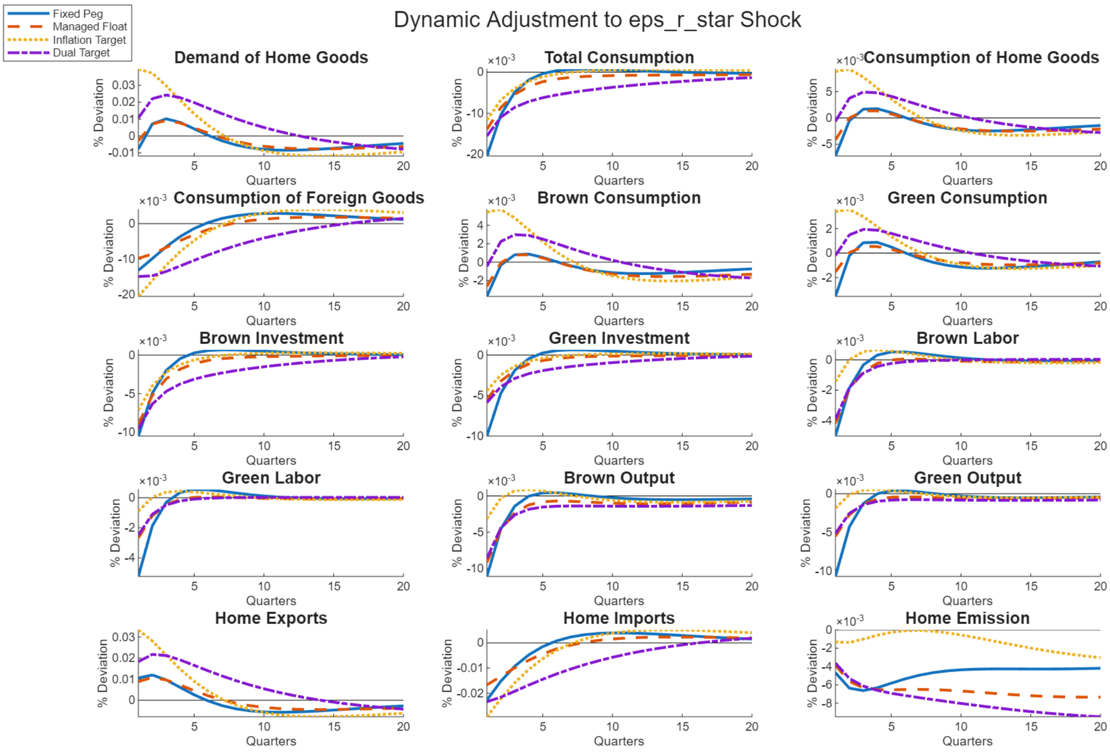


Figure 8: Impulse Responses of Different Monetary and Exchange Rate Regimes: Contractionary Foreign Monetary Policy Shock

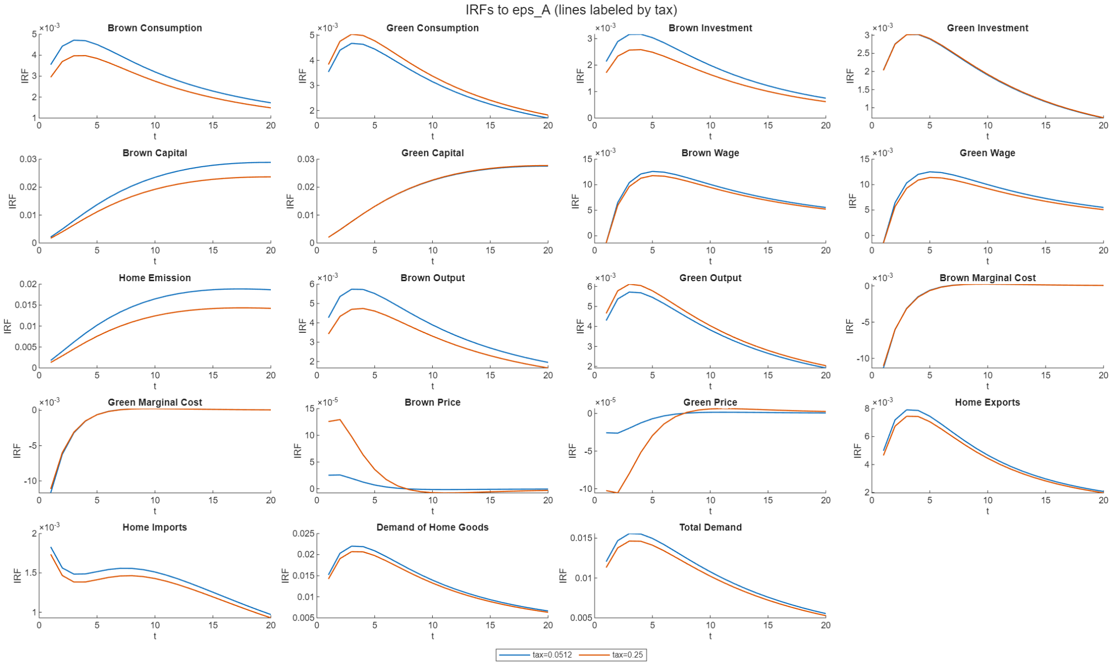


Figure 9: Impulse Responses of Fixed Exchange Rate Regime: Positive TFP Shock

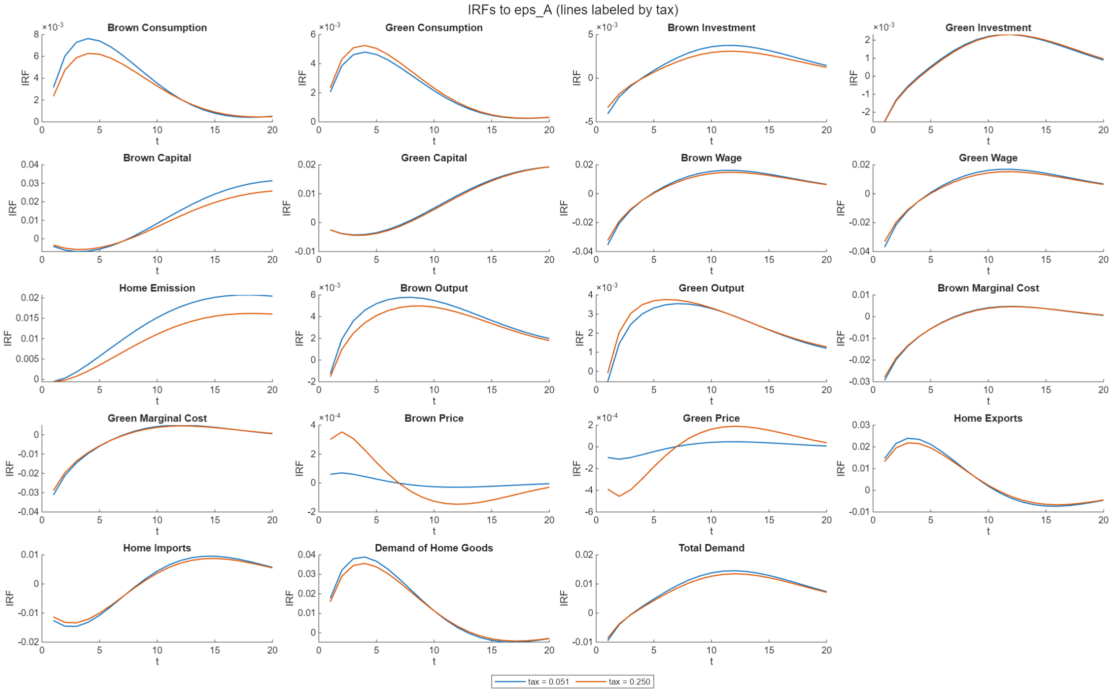


Figure 10: Impulse Responses of Managed Float Exchange Rate Regime: Positive TFP Shock

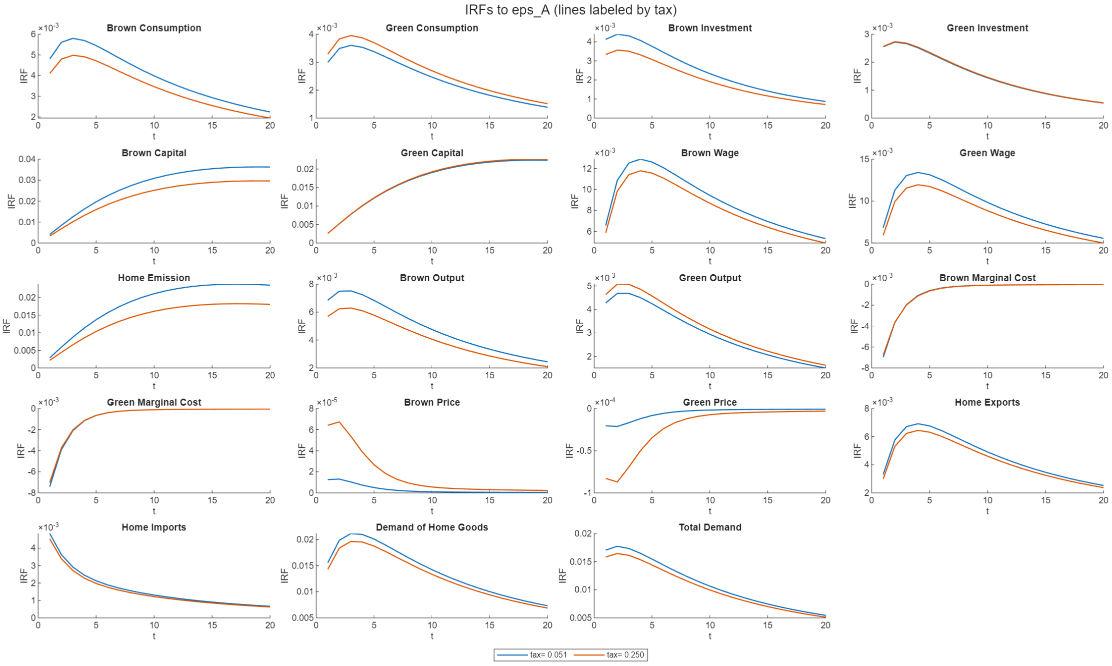


Figure 11: Impulse Responses of Inflation Targeting Regime: Positive TFP Shock

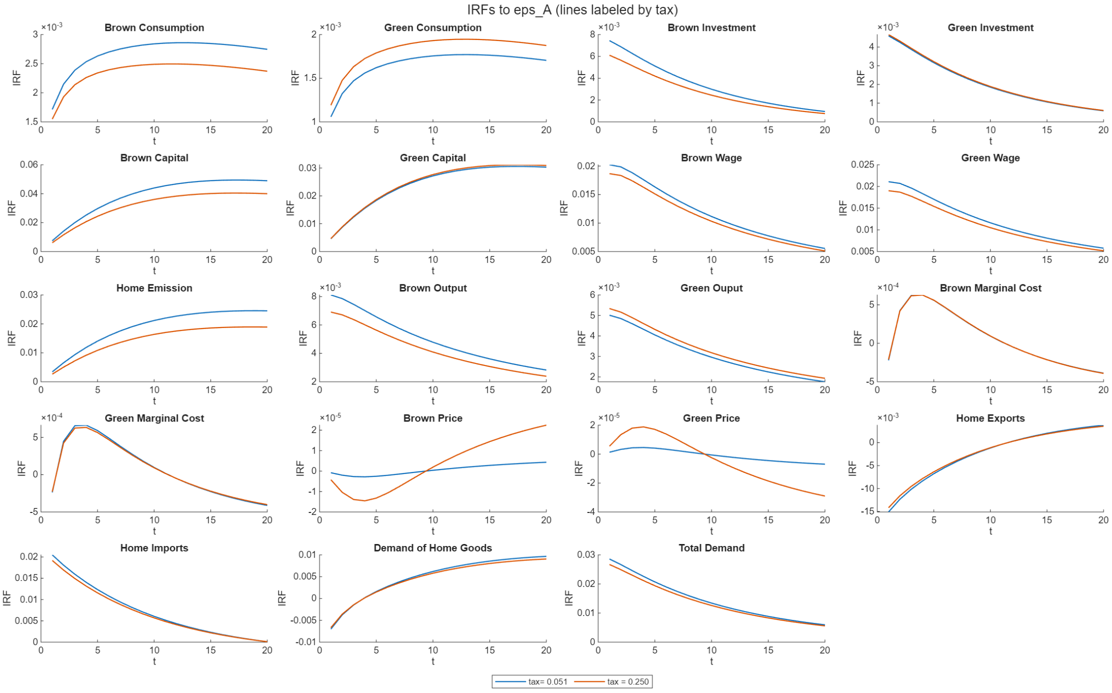


Figure 12: Impulse Responses of Dual Targeting Regime: Positive TFP Shock

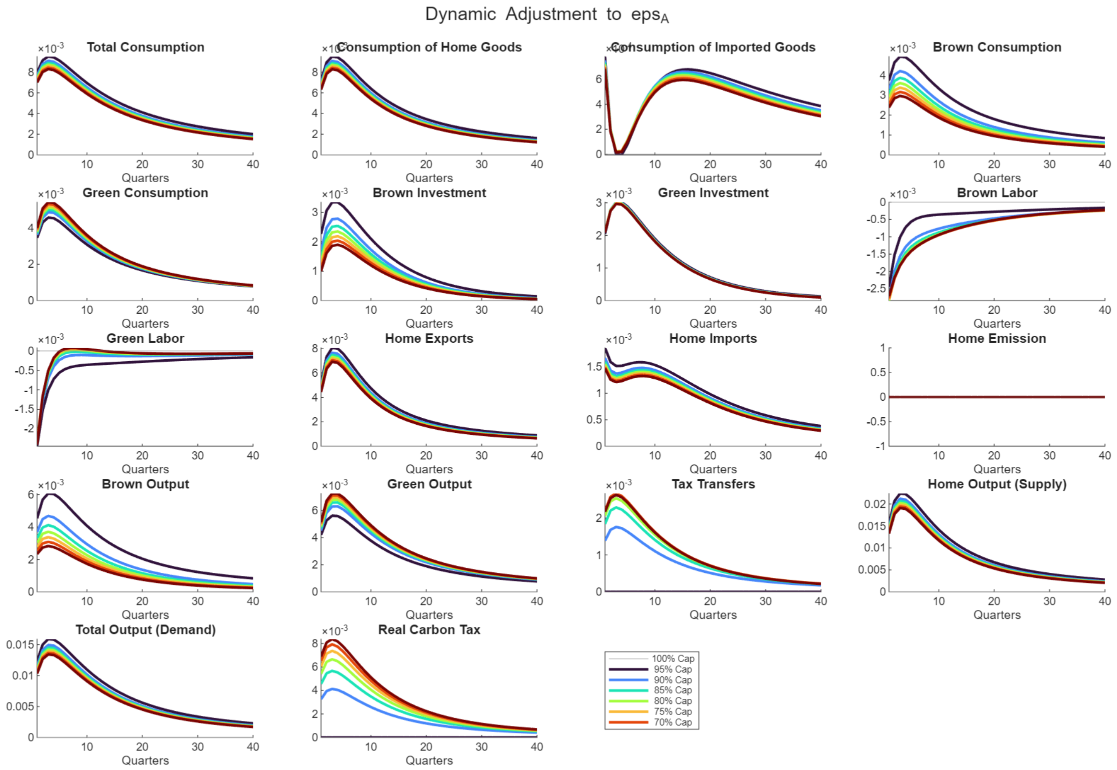


Figure 13: **Cap-and-Trade Policy**:Impulse Responses of Different Carbon Tax Rates: Positive TFP Shock

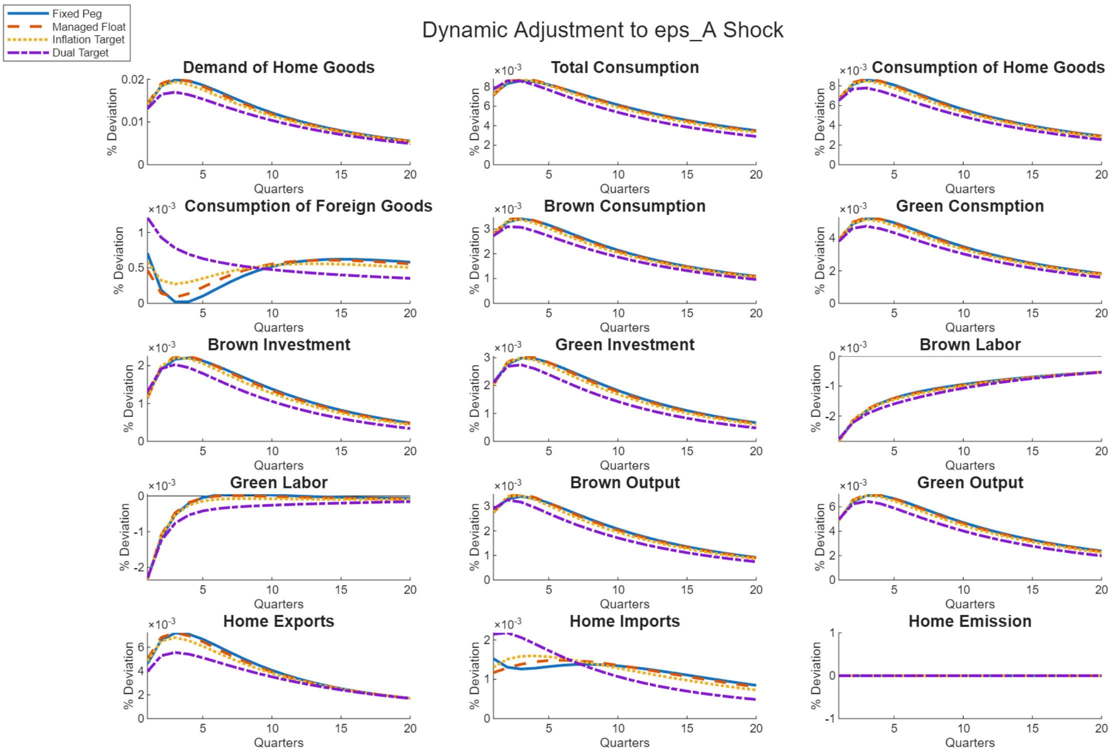


Figure 14: **Cap-and-Trade Policy (90% of total emission)** :Impulse Responses of Different Monetary Policy Regimes: Positive TFP Shock

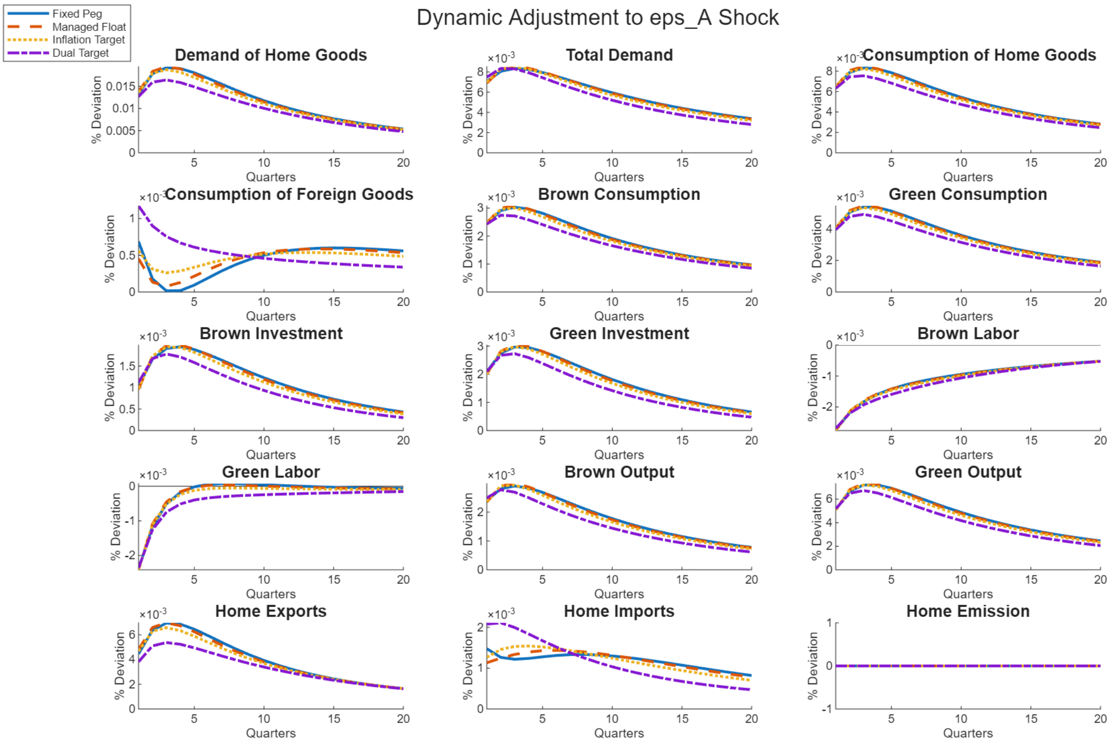


Figure 15: **Cap-and-Trade Policy (80% of total emission)** :Impulse Responses of Different Monetary Policy Regimes: Positive TFP Shock

Tables

Table 1: Baseline calibration

Param.	Value	Description	Reference
Preferences			
β	0.99	Subjective discount factor (quarterly)	Smets and Wouters (2007)
ψ_C	1.5	Coefficient of relative risk aversion	Galí (2015)
ψ_L	1.0	Inverse of the Frisch elasticity	Annicchiarico and Di Dio (2015)
ω_C	0.6	Home bias	Beltran and Draper (2008)
ω_d	0.5	Domestic expenditure share on brown goods	Minesso and Pagliari (2023)
η_C	1.5	Elasticity of substitution between home and imported consumption	Bajzik et al. (2020)
σ_d	2	Elasticity of substitution between brown and green goods	Acemoglu et al. (2012)
ω_L	0.5	Pins down steady-state labor hours in brown sector	Bridgman et al. (2018)
ν_L	2	Elasticity of substitution between brown and green labor hours	Ngai and Pissarides (2008)
Technology and investment			
δ	0.025	Depreciation rate of capital	Cooley (1995)
α	0.37	Capital share in production	Minesso and Pagliari (2023)
γ_I	15	Investment adjustment cost	Annicchiarico and Di Dio (2015)
Price setting			
ρ	6	Elasticity of substitution across varieties	Annicchiarico and Di Dio (2015)
ω	0.5	Calvo probability of not changing price	Minesso and Pagliari (2023)
ξ_h	0	Inflation indexation parameter	No indexation
Monetary policy (Taylor rule)			
ρ_r	0.74	Interest-rate smoothing parameter	Smets and Wouters (2007)
ϕ_π	1.5	Inflation response coefficient	MacDonald and Taylor (1993)
ϕ_y	0.1	Output response coefficient	Annicchiarico and Di Dio (2015)
Environmental policy and small open economy			

Continued on next page.

Table 1 continued.

Param.	Value	Description	Reference
ρ_m	0.9	Persistence of emissions	Minesso and Pagliari (2023)
d_0	1.3950e-3	Coefficient in climate damage function	Annicchiarico and Di Dio (2015)
d_1	-6.6722e-6	Emissions scaling parameter (level)	Heutel (2012)
d_2	1.4647e-8	Emissions parameter (curvature)	Heutel (2012)
ε_1	1.17	Scale in abatement cost function	Angelopoulos et al. (2010)
ε_2	2.8	Curvature in abatement cost function	Angelopoulos et al. (2010)
ζ	0.45	Emissions per unit of output	Annicchiarico and Di Dio (2015)
τ	0.25	Carbon tax rate	Subject to changes
ψ_b	0.01	Risk-premium elasticity w.r.t. net foreign assets (NFA)	Schmitt-Grohé and Uribe (2003)
ρ_π^*	0.7	Persistence of foreign inflation	Justiniano and Preston (2010)
ρ_r^*	0.9	Persistence of foreign interest rate	Justiniano and Preston (2010)
η_e	1.5	Price elasticity of export demand	Bauer and Swanson (2023)
ω_e	0.2	Export demand shifter	Schmitt-Grohé and Uribe (2003)
ρ_y^*	0.8	Persistence of foreign (world) demand	Justiniano and Preston (2010)
Innovations and exogenous processes			
ρ_A	0.9	Persistence of technology	Galí (2015)
σ_A	0.01	Std. dev. of technology innovation ϵ_t^A	Smets and Wouters (2007)
σ_r	0.01	Std. dev. of monetary policy innovation ϵ_t^r	Smets and Wouters (2007)
σ_{r^*}	0.01	Std. dev. of foreign monetary policy innovation $\epsilon_t^{r^*}$	Schmitt-Grohé and Uribe (2003)

Table 2: Welfare Analysis (deterministic): Fixed Exchange Rate Regime

Tax Rate (τ)	**Steady-state Welfare (Utils)**	**Cons. Equiv. Change (%)**
0.00	9.6581	0.0000
0.05	9.0761	-0.7117
0.13	8.1222	-1.8618
0.22	6.9683	-3.2266
0.25	6.572	-3.6888
0.30	5.9013	-4.4634
0.39	4.6692	-5.8622

Table 3: Welfare Analysis Results (deterministic): Managed Float Exchange Rate Regime

Tax Rate (τ)	**Steady-state Welfare (Utils)**	**Cons. Equiv. Change (%)**
0.00	9.6581	0.0000
0.05	9.0761	-0.7117
0.13	8.1222	-1.8618
0.22	6.9683	-3.2266
0.25	6.572	-3.6888
0.30	5.9013	-4.4634
0.39	4.6692	-5.8622

Table 4: Welfare Analysis Results (deterministic): Inflation Targeting Regime

Tax Rate (τ)	**Steady-state Welfare (Utils)**	**Cons. Equiv. Change (%)**
0.00	9.6581	0.0000
0.05	9.0761	-0.7117
0.13	8.1222	-1.8618
0.22	6.9683	-3.2266
0.25	6.572	-3.6888
0.30	5.9013	-4.4634
0.39	4.6692	-5.8622

Table 5: Welfare Analysis Results (deterministic): Dual Targeting Regime

Tax Rate (τ)	**Steady-state Welfare (Utils)**	**Cons. Equiv. Change (%)**
0.00	9.6581	0.0000
0.05	9.0761	-0.7117
0.13	8.1222	-1.8618
0.22	6.9683	-3.2266
0.25	6.572	-3.6888
0.30	5.9013	-4.4634
0.39	4.6692	-5.8622

Table 6: Dynamic Welfare Analysis: Fixed Exchange Rate Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.00	-190.62	0.56507	0.00000
0.05	-191.40	0.55462	-0.40659
0.13	-192.28	0.60317	-0.87034
0.22	-193.42	0.46093	-1.46670
0.25	-193.83	0.55930	-1.68180
0.30	-194.53	0.63545	-2.05150
0.39	-195.86	0.53745	-2.74790

Note: Welfare is calculated using second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the welfare gain/loss relative to the baseline ($\tau = 0$) expressed as a percentage of permanent consumption.

Table 7: Dynamic Welfare Analysis: Managed Float Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.00	-190.75	0.4623	0.0000
0.05	-191.46	0.4843	-0.3722
0.13	-192.38	0.5422	-0.8557
0.22	-193.49	0.4125	-1.4396
0.25	-193.94	0.5016	-1.6715
0.30	-194.67	0.5813	-2.0558
0.39	-195.94	0.4835	-2.7240

Note: Welfare ($E[W]$) is the ergodic mean from second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the percentage of permanent consumption a household would be willing to pay (negative) or receive (positive) to move from the $\tau = 0$ baseline to the alternative tax rate.

Table 8: Dynamic Welfare Analysis: Inflation Targeting Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.00	-190.27	0.5560	0.0000
0.05	-191.01	0.5431	-0.3908
0.13	-191.91	0.5993	-0.8624
0.22	-193.05	0.4743	-1.4616
0.25	-193.47	0.5615	-1.6846
0.30	-194.19	0.6271	-2.0629
0.39	-195.49	0.5348	-2.7479

Note: Welfare ($E[W]$) is the ergodic mean from second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the percentage of permanent consumption a household would be willing to pay (negative) or receive (positive) to move from the $\tau = 0$ baseline to the alternative tax rate.

Table 9: Dynamic Welfare Analysis: Dual Targeting Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.00	-190.59	0.4851	0.0000
0.05	-191.31	0.4968	-0.3770
0.13	-192.23	0.5526	-0.8573
0.22	-193.35	0.4266	-1.4460
0.25	-193.78	0.5139	-1.6748
0.30	-194.51	0.5880	-2.0570
0.39	-195.80	0.4938	-2.7300

Note: Welfare ($E[W]$) is the ergodic mean from second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the percentage of permanent consumption a household would be willing to pay (negative) or receive (positive) to move from the $\tau = 0$ baseline to the alternative tax rate.

Table 10: Dynamic Welfare of Fixed Exchange Rate Regime Across Emission Caps (Baseline: 100% Cap)

Cap (% of Baseline)	**Mean Welfare**	**Welfare Volatility**	**Cons. Equiv. Var. (%)**
100	-191.28	0.50121	0.00000
95	-191.86	0.48474	-0.30193
90	-192.27	0.51419	-0.51636
85	-192.81	0.39519	-0.79893
80	-193.45	0.47002	-1.13670
75	-194.21	0.52820	-1.52930
70	-195.10	0.45128	-1.99700

Note: Mean Welfare represents the stochastic mean ($E[W_t]$) obtained from a second-order approximation. Welfare Volatility is the standard deviation of welfare. Consumption Equivalent Variation (CEV) represents the percentage of lifetime consumption the household would lose (negative) relative to the 100% Baseline scenario.

Table 11: Dynamic Welfare of Managed Float Regime Across Emission Caps (Baseline: 100% Cap)

Cap (% of Baseline)	**Mean Welfare**	**Welfare Volatility**	**Cons. Equiv. Var. (%)**
100	-193.69	0.48334	0.00000
95	-194.18	0.50807	-0.24878
90	-194.55	0.51022	-0.44262
85	-195.17	0.49018	-0.76419
80	-195.82	0.51349	-1.10010
75	-196.70	0.46730	-1.55380
70	-197.34	0.50508	-1.88370

Note: Mean Welfare represents the stochastic mean ($E[W_t]$) obtained from a second-order approximation. Welfare Volatility is the standard deviation of welfare. Consumption Equivalent Variation (CEV) represents the percentage of lifetime consumption the household would lose (negative) relative to the 100% Baseline scenario.

Table 12: Dynamic Welfare of Inflation Targeting Regime Across Emission Caps (Baseline: 100% Cap)

Cap (% of Baseline)	**Mean Welfare**	**Welfare Volatility**	**Cons. Equiv. Var. (%)**
100	-183.34	0.41328	0.00000
95	-183.88	0.40267	-0.29601
90	-184.30	0.44053	-0.52381
85	-184.85	0.40570	-0.82739
80	-185.44	0.41222	-1.14500
75	-186.28	0.34779	-1.60590
70	-186.91	0.43702	-1.94920

Note: Mean Welfare represents the stochastic mean ($E[W_t]$) obtained from a second-order approximation. Welfare Volatility is the standard deviation of welfare. Consumption Equivalent Variation (CEV) represents the percentage of lifetime consumption the household would lose (negative) relative to the 100% Baseline scenario.

Table 13: Dynamic Welfare of Dual Targeting Regime Across Emission Caps (Baseline: 100% Cap)

Cap (% of Baseline)	**Mean Welfare**	**Welfare Volatility**	**Cons. Equiv. Var. (%)**
100	-185.90	0.28970	0.00000
95	-186.27	0.29553	-0.20217
90	-186.80	0.31343	-0.48308
85	-187.33	0.30084	-0.77018
80	-187.92	0.33880	-1.08690
75	-188.59	0.27652	-1.45020
70	-189.43	0.34759	-1.89960

Note: Mean Welfare represents the stochastic mean ($E[W_t]$) obtained from a second-order approximation. Welfare Volatility is the standard deviation of welfare. Consumption Equivalent Variation (CEV) represents the percentage of lifetime consumption the household would lose (negative) relative to the 100% Baseline scenario.

Table 14: Dynamic Welfare Analysis: Fixed Exchange Rate Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.000	-190.62	0.5651	0.0000
0.085	-191.81	0.5557	-0.6236
0.124	-192.20	0.6030	-0.8313
0.168	-192.74	0.4595	-1.1143
0.217	-193.39	0.5582	-1.4539
0.271	-194.14	0.6343	-1.8472
0.332	-195.06	0.5354	-2.3301

Note: Welfare ($E[W]$) is the ergodic mean from second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the percentage of permanent consumption a household would be willing to pay (negative) or receive (positive) to move from the $\tau = 0$ baseline to the alternative tax rate.

Table 15: Dynamic Welfare Analysis: Managed Float Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.000	-190.75	0.4623	0.0000
0.085	-191.87	0.4847	-0.5906
0.124	-192.30	0.5422	-0.8164
0.168	-192.82	0.4118	-1.0851
0.217	-193.50	0.5010	-1.4423
0.271	-194.28	0.5807	-1.8503
0.332	-195.14	0.4824	-2.3044

Note: Welfare ($E[W]$) is the ergodic mean from second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the percentage of permanent consumption a household would be willing to pay (negative) or receive (positive) to move from the $\tau = 0$ baseline to the alternative tax rate.

Table 16: Dynamic Welfare Analysis: Inflation Targeting Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.000	-190.27	0.5560	0.0000
0.085	-191.43	0.5440	-0.6093
0.124	-191.83	0.5992	-0.8231
0.168	-192.37	0.4729	-1.1074
0.217	-193.04	0.5605	-1.4555
0.271	-193.80	0.6260	-1.8576
0.332	-194.70	0.5329	-2.3286

Note: Welfare ($E[W]$) is the ergodic mean from second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the percentage of permanent consumption a household would be willing to pay (negative) or receive (positive) to move from the $\tau = 0$ baseline to the alternative tax rate.

Table 17: Dynamic Welfare Analysis: Dual Targeting Regime

Tax Rate (τ)	**Mean Welfare ($E[W]$)**	**Volatility**	**CEV vs Baseline (%)**
0.000	-190.59	0.4851	0.0000
0.085	-191.73	0.4973	-0.5954
0.124	-192.15	0.5525	-0.8180
0.168	-192.67	0.4257	-1.0916
0.217	-193.35	0.5132	-1.4456
0.271	-194.12	0.5874	-1.8515
0.332	-195.00	0.4924	-2.3105

Note: Welfare ($E[W]$) is the ergodic mean from second-order stochastic simulations. CEV (Consumption Equivalent Variation) represents the percentage of permanent consumption a household would be willing to pay (negative) or receive (positive) to move from the $\tau = 0$ baseline to the alternative tax rate.

Appendix Contents

A Additional Tables & Figures

B A System of Equilibrium Conditions

This appendix reports the model equations used to solve the DSGE model in Dynare.

We listed all first order conditions and other equilibrium conditions that we used to solve the model. The conditions are derived from household utility maximization problem, intermediate goods firms' profit maximization problems, retailers' optimal pricing problem, final goods producers' profit maximization problem, and market clearing conditions.

Appendix B reports the complete system of equilibrium conditions corresponding to the model described in Section 3, using identical notation and timing conventions.

Intra-temporary condition for optimal consumption. Household Consumption:

$$\lambda_{h,t} = \frac{C_{h,t}^{-\psi_C}}{P_{h,t}} \tag{B.1}$$

$$C_{d,t} = [(\omega_d)^{\frac{1}{\sigma_d}} C_{h,b,t}^{\frac{\sigma_d - 1}{\sigma_d}} + (1-\omega_d)^{\frac{1}{\sigma_d}} C_{h,g,t}^{\frac{\sigma_d - 1}{\sigma_d}}]^{\frac{\sigma_d}{\sigma_d - 1}} \tag{B.2}$$

$$C_{h,t} = (\omega_C^{\frac{1}{\eta_C}} C_{d,t}^{\frac{\eta_C - 1}{\eta_C}} + (1-\omega_C)^{\frac{1}{\eta_C}} C_{m,t}^{\frac{\eta_C - 1}{\eta_C}})^{\frac{\eta_C}{\eta_C - 1}} \tag{B.3}$$

$$\frac{C_{h,b,t}}{C_{h,g,t}} = (\frac{\omega_d}{1-\omega_d})^{\sigma_d} (\frac{P_{h,b,t}}{P_{h,g,t}})^{-\sigma_d} \tag{B.4}$$

$$\frac{C_{d,t}}{C_{m,t}} = (\frac{\omega_C}{1-\omega_C})^{\eta_C} (\frac{P_{d,t}}{P_{m,t}})^{-\eta_C} \tag{B.5}$$

$$P_{m,t} = S_t P_t^* \tag{B.6}$$

Labor FOCs:

$$\phi_L L_{h,t}^{\psi_L} \omega_L (\frac{L_{h,b,t}}{L_{h,t}})^{\rho_L - 1} = \lambda_{h,t} W_{h,b,t} \tag{B.7}$$

$$\phi_L L_{h,t}^{\psi_L} (1-\omega_L) (\frac{L_{h,g,t}}{L_{h,t}})^{\rho_L - 1} = \lambda_{h,t} W_{h,g,t} \tag{B.8}$$

Inter-temporary Euler equations:

$$1 = \beta\,\mathbb{E}_t\left[R_{h,t}\left(\frac{\lambda_{h,t+1}}{\lambda_{h,t}}\right)\right] \tag{B.9}$$

$$1 = \beta\mathbb{E}_t[\frac{\lambda_{h,t+1}}{\lambda_{h,t}}\frac{S_{t+1}}{S_t}R_t^* exp(\psi_b)(\frac{S_t B_{h,t}^*}{P_{h,t}Y_{h,t}} - \bar{B}^*] \tag{B.10}$$

Investment FOCs: Tobin's q (definition)

$$q_{h,b,t} = \lambda_{h,t}P_{h,t}[1 + \gamma_I(\frac{I_{h,b,t}}{K_{h,b,t}} - \delta)] \tag{B.11}$$

$$q_{h,g,t} = \lambda_{h,t}P_{h,t}[1 + \gamma_I(\frac{I_{h,g,t}}{K_{h,g,t}} - \delta)] \tag{B.12}$$

q's Capital Euler equations

$$q_{h,b,t} = \beta\mathbb{E}_t[\lambda_{h,t+1}(R_{h,b,t+1}^K + P_{h,t+1}\frac{\gamma_I}{2}((\frac{I_{h,b,t+1}}{K_{h,b,t+1}})^2 - \delta^2) + (1-\delta)q_{h,b,t+1}] \tag{B.13}$$

$$q_{h,g,t} = \beta\mathbb{E}_t[\lambda_{h,t+1}(R_{h,g,t+1}^K + P_{h,t+1}\frac{\gamma_I}{2}((\frac{I_{h,g,t+1}}{K_{h,g,t+1}})^2 - \delta^2) + (1-\delta)q_{h,g,t+1}] \tag{B.14}$$

Capital accumulation equations

$$K_{h,b,t+1} = I_{h,b,t} + (1-\delta)\,K_{h,b,t} \tag{B.15}$$

$$K_{h,g,t+1} = I_{h,g,t} + (1-\delta)\,K_{h,g,t} \tag{B.16}$$

Output after the loss from emission

$$\mathcal{L}(M_{h,t}) = d_0 + d_1 M_{h,t} + d_2 M_{h,t}^2 \tag{B.17}$$

$$\omega_h\mathcal{L}(M_t^*) = \omega_h(d_0 + d_1 M_t^* + d_2(M_t^*)^2) \tag{B.18}$$

$$V_{h,t} = \mathcal{L}(M_{h,t}) + \omega_h \mathcal{L}(M_t^*) \tag{B.19}$$

Emission evolution function

$$M_{h,t} = \rho_m \, M_{h,t-1} + (1 - \kappa_{h,b,t})\zeta X_{h,b,t} \tag{B.20}$$

$$\log(M_t^*) = (1 - \rho_m^*) \, \log(\bar{M}^*) + \rho_m^* \log(M_{t-1}^*) + \epsilon_{m,t}^* \tag{B.21}$$

Technology

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A) \log A_{ss} + \epsilon_t^A \tag{B.22}$$

Abatement FOC

$$\varepsilon_1 \varepsilon_2 \, \kappa_{h,b,t}^{\varepsilon_2 - 1} = \zeta \, \tau \tag{B.23}$$

Profit Maximization of brown firms

$$\frac{K_{h,b,t}}{L_{h,b,t}} = \frac{\alpha}{1-\alpha} \frac{W_{h,b,t}}{R_{h,b,t}^K} \tag{B.24}$$

Nominal Marginal Cost of brown firms (cost minimization)

$$MC_{h,b,t} = \frac{\left(W_{h,b,t}\right)^{1-\alpha} \left(R_{h,b,t}^K\right)^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha} \, A_t \, (1 - V_{h,t})} + P_{h,t} \varepsilon_1 \kappa_{h,b,t}^{\varepsilon_2} + (1 - \kappa_{h,b,t})\zeta P_{z,t} \tag{B.25}$$

Zero-profit Condition

$$P_{x,b,t} = MC_{h,b,t} \tag{B.26}$$

Real Marginal Cost

$$mc_{h,b,t} = \frac{P_{x,b,t}}{P_{h,b,t}} \tag{B.27}$$

Profit Maximization of green firms

$$\frac{K_{h,g,t}}{L_{h,g,t}} = \frac{\alpha \psi_g}{1-\alpha} \frac{W_{h,g,t}}{R_{h,g,t}^K} \tag{B.28}$$

Nominal Marginal Cost of green firms (cost minimization)

$$MC_{h,g,t} = \frac{W_{h,g,t}^{1-\alpha} (R_{h,g,t}^K)^{\alpha}}{(1 - V_{h,t}) A_t \psi_g^{\alpha} (1-\alpha)^{1-\alpha} \alpha^{\alpha}} \tag{B.29}$$

Zero-profit Condition

$$P_{x,g,t} = MC_{h,g,t} \tag{B.30}$$

Real Marginal Cost

$$mc_{h,g,t} = \frac{P_{x,g,t}}{P_{h,g,t}} \tag{B.31}$$

Calvo $S1/S2$

$$S1_{h,b,t} = Z_{h,b,t}\, P^{\rho}_{h,b,t}\, mc_{h,b,t} + \beta\, \omega\, \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \Pi^{\rho}_{h,b,t+1}\, S1_{h,b,t+1} \right] \tag{B.32}$$

$$S2_{h,b,t} = Z_{h,b,t}\, P^{\rho-1}_{h,b,t} + \beta\, \omega\, \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \Pi^{\rho-1}_{h,b,t+1}\, S2_{h,b,t+1} \right] \tag{B.33}$$

Optimal reset price

$$P^{opt}_{h,b,t} = \left(\frac{S1_{h,b,t}}{S2_{h,b,t}} \right) \left(\frac{\rho}{\rho - 1} \right) \tag{B.34}$$

Calvo $S1/S2$

$$S1_{h,g,t} = Z_{h,g,t}\, P^{\rho}_{h,g,t}\, mc_{h,g,t} + \beta\, \omega\, \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \Pi^{\rho}_{h,g,t+1}\, S1_{h,g,t+1} \right] \tag{B.35}$$

$$S2_{h,g,t} = Z_{h,g,t}\, P^{\rho-1}_{h,g,t} + \beta\, \omega\, \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \Pi^{\rho-1}_{h,g,t+1}\, S2_{h,g,t+1} \right] \tag{B.36}$$

Optimal reset price

$$P^{opt}_{h,g,t} = \left(\frac{S1_{h,g,t}}{S2_{h,g,t}} \right) \left(\frac{\rho}{\rho - 1} \right) \tag{B.37}$$

$$y_{h,b,t+s|t}(i) = [\frac{P^{\text{opt}}_{h,b,t}}{P_{h,b,t+s}}]^{-\rho} Z_{h,b,t+s} \tag{B.38}$$

$$y_{h,g,t+s|t}(j) = [\frac{P^{\text{opt}}_{h,g,t}}{P_{h,g,t+s}}]^{-\rho} Z_{h,g,t+s} \tag{B.39}$$

Aggregation in brown and green retail sectors

$$Z_{h,b,t} = (\int_0^1 (y_{h,b,t}(i))^{\frac{\rho-1}{\rho}} di)^{\frac{\rho}{\rho-1}} \tag{B.40}$$

$$Z_{h,g,t} = (\int_0^1 (y_{h,g,t}(i))^{\frac{\rho-1}{\rho}} dj)^{\frac{\rho}{\rho-1}} \tag{B.41}$$

$$P_{h,b,t} = ((1-\omega)(P^{\text{opt}}_{h,b,t})^{1-\rho} + \omega P^{1-\rho}_{h,b,t-1})^{\frac{1}{1-\rho}} \tag{B.42}$$

$$P_{h,g,t} = ((1-\omega)(P^{\text{opt}}_{h,g,t})^{1-\rho} + \omega P^{1-\rho}_{h,g,t-1})^{\frac{1}{1-\rho}} \tag{B.43}$$

Final good producer

$$Z_{h,b,t} = \omega_Y (\frac{P_{h,b,t}}{P_{d,t}})^{-\eta} Y_{h,t} \tag{B.44}$$

$$Z_{h,g,t} = (1-\omega_Y)(\frac{P_{h,g,t}}{P_{d,t}})^{-\eta} Y_{h,t} \tag{B.45}$$

define the final good price index as:

$$P_{d,t} = [\omega_Y P^{1-\eta}_{h,b,t} + (1-\omega_Y) P^{1-\eta}_{h,g,t}]^{\frac{1}{1-\eta}} \tag{B.46}$$

$$P_{h,t} = (\omega_C P^{1-\eta_C}_{d,t} + (1-\omega_C) P^{1-\eta_C}_{m,t})^{\frac{1}{1-\eta_C}} \tag{B.47}$$

$$Y_{h,t} = \omega_Y (\frac{P_{d,t}}{P_{h,t}})^{-\eta} Y_t \tag{B.48}$$

$$Y^*_t = (1-\omega_Y)(\frac{P_{m,t}}{P_{h,t}})^{-\eta} Y_t \tag{B.49}$$

$$P_{m,t} = S_t P^*_t \tag{B.50}$$

$$\log(\Pi^*_t) = \rho^*_\pi \log(\Pi^*_{t-1}) + (1-\rho^*_\pi)\log(\bar{\Pi}^*) + \epsilon^*_{\pi,t} \tag{B.51}$$

$$P^*_t = \Pi^*_t P^*_{t-1} \tag{B.52}$$

Inflation

$$\Pi_{h,t} = \frac{P_{h,t}}{P_{h,t-1}} \tag{B.53}$$

$$\log(R_t^*) = \rho_r^* \log(R_{t-1}^*) + (1-\rho_r^*)\log(\bar{R}^*) + \epsilon_{r,t}^* \tag{B.54}$$

$$D_{h,b,t} = (1-\omega)(\frac{P_{h,b,t}^{opt}}{P_{h,b,t}})^{-\rho} + \omega(\Pi_{h,b,t})^{\rho} D_{h,b,t-1} \tag{B.55}$$

$$D_{h,g,t} = (1-\omega)(\frac{P_{h,g,t}^{opt}}{P_{h,g,t}})^{-\rho} + \omega(\Pi_{h,g,t})^{\rho} D_{h,g,t-1} \tag{B.56}$$

$$Z_{h,b,t} = \frac{y_{h,b,t}}{D_{h,b,t}} \tag{B.57}$$

$$Z_{h,g,t} = \frac{y_{h,g,t}}{D_{h,g,t}} \tag{B.58}$$

$$\pi_{h,b,t}^{ret} = P_{h,b,t} Z_{h,b,t} - P_{x,b,t} D_{h,b,t} Z_{h,b,t} \tag{B.59}$$

$$\pi_{h,g,t}^{ret} = P_{h,g,t} Z_{h,g,t} - P_{x,g,t} D_{h,g,t} Z_{h,g,t} \tag{B.60}$$

$$\log(R_{h,t}) = \log(R_t^*) + \psi_b(\frac{S_t B_{h,t}^*}{P_{h,t} Y_{h,t}} - \bar{b}^*) + \mathbb{E}_t(\log(S_{t+1}) - \log(S_t)) \tag{B.61}$$

$$RE_{h,t} = P_{zt}(1-\kappa_{h,b,t})\zeta(X_{h,b,t}) \tag{B.62}$$

$$T_{h,t} = RE_{h,t} \tag{B.63}$$

$$N_{h,t} = Y_t + NX_{h,t} \tag{B.64}$$

$$\begin{aligned} Y_t = {} & C_{h,t} + I_{h,b,t} + I_{h,g,t} \\ & + \frac{\gamma_I}{2}\left(\frac{I_{h,b,t}}{K_{h,b,t}} - \delta\right)^2 K_{h,b,t} + \frac{\gamma_I}{2}\left(\frac{I_{h,g,t}}{K_{h,g,t}} - \delta\right)^2 K_{h,g,t} \\ & + \varepsilon_1\, \kappa_{h,b,t}^{\varepsilon_2} X_{h,b,t}. \end{aligned} \tag{B.65}$$

Resource constraint

$$NX_{h,t} = \mathcal{E}_{h,t} - \mathcal{I}_{h,t} \tag{B.66}$$

$$\mathcal{I}_{h,t} = Y_{m,t} \tag{B.67}$$

$$Y_{m,t} = (1 - \omega_y)(\frac{P_{m,t}}{P_{h,t}})^{-\eta_y} Y_t \tag{B.68}$$

$$P_{m,t} = S_t P_t^* \tag{B.69}$$

$$\mathcal{E}_{h,t} = \omega_e(\frac{P_{e,t}^*}{P_t^*})^{-\eta_e} Y_{f,t}^* \tag{B.70}$$

$$P_{e,t}^* = \frac{P_{d,t}}{S_t} \tag{B.71}$$

$$\log(Y_{f,t}^*) = (1 - \rho_{y,t}^*) \log(\bar{Y}_f^*) + \rho_{y,t}^* \log(Y_{f,t-1}^*) + \epsilon_{f,t}^* \tag{B.72}$$

$$S_t B_{h,t}^* - S_t R_{t-1}^* exp(\psi_b(\frac{S_{t-1} B_{h,t-1}^*}{P_{h,t-1} Y_{h,t-1}})) = P_{d,t}\mathcal{E}_{h,t} - P_{m,t}\mathcal{I}_{h,t} \tag{B.73}$$